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DYNAMICS OF A HIGHER ORDER NONLINEAR RATIONAL DIFFERENCE EQUATION

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Abstract. In this paper, we discuss some qualitative properties of the positive solutions to the higher order rational nonlinear difference equation $x_{n+1} = \dfrac{\alpha x_{n-m}}{\beta + \gamma\left(\prod_{i=1}^{N} x_{n-k_i}\right)\left(\sum_{i=1}^{N} x_{n-k_i}\right)}$, $n = 0, 1, 2, ...$ where the parameters $\alpha, \beta, \gamma \in (0, \infty)$, while m, k_i,(i=1,2,...,N) are positive integers, such that $m < k_1 < k_2 < ... < k_N$. The initial conditions $x_{-m}, ..., x_{-k_1}, ..., x_{-k_2}, ..., x_{-k_N}, ...,x_{-1}, x_0$ are arbitrary positive real numbers.

Keywords. Difference equations; Rational difference equations; qualitative properties of solutions of difference equations; Equilibrium points.

AMS (MOS) subject classification: 39A10; 39A11; 39A99;34C99.

1 Introduction

There has been a great interest in studying the global attractivity, the boundedness character and the periodicity nature of nonlinear difference equations. The study of these equations is challenging and rewarding and is still in its infancy. We believe that the nonlinear rational difference equations are of importance in their own right. Furthermore the results about such equations offer prototypes for the development of the basic theory of the global behavior of nonlinear difference equations.
The objective of this article is to investigate some qualitative behavior of the solutions of the nonlinear difference equation.

$$x_{n+1} = \frac{\alpha x_{n-m}}{\beta + \gamma\left(\prod_{i=1}^{N} x_{n-k_i}\right)\left(\sum_{i=1}^{N} x_{n-k_i}\right)}, \quad n = 0, 1, 2, ... \tag{1}$$

where the parameters $\alpha, \beta, \gamma \in (0, \infty)$, while m, k_i $(i = 1, 2, ...N)$ are positive integers, such that $m < k_1 < k_2 < ... < k_N$. The initial conditions $x_{-m}, ..., x_{-k_1}, ..., x_{-k_2}, ..., x_{-k_N}, ...,$ k_{-1}, x_0 are arbitrary positive real numbers. Eq. (1) has been discussed in [10] when $N = 2, k_1 = 2$ and $k_2 = 4$ and in [53] when $N = 2, k_1 = k$ and $k_2 = l$, where some global behavior of these rational difference equations have been discussed. In order to study the global behavior of the higher order rational difference equation (1), we need the following well-known definitions and results [1-53].