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EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR ABSTRACT TWO-POINT BOUNDARY VALUE PROBLEMS

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Abstract. In this paper the existence and uniqueness of solutions for a class of abstract two-point boundary value problems are discussed. The solvability for these boundary value problems has important applications in control theory, economics, mathematical finance, and many other areas. Using homotopy technique and fixed point theorems the existence and uniqueness of solutions for boundary value problems are established under monotonic conditions. In addition, several examples are provided to illustrate the application of the obtained results.

Keywords. Existence and uniqueness, solvability, boundary value problems, fixed-point theorem, optimal control.

AMS (MOS) subject classification: 34B15, 34G20, 49K15.

1 Introduction

In this paper we investigate the existence and uniqueness of solutions for the following two-point boundary value problem (BVP) on $[a,b]$:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + F(x(t), N(t)p(t), t),\ x(a) = x_0, \\ \dot{p}(t) &= -A^*(t)p(t) + G(x(t), p(t), t),\ p(b) = \xi(x(b)). \end{aligned} \tag{1.1}$$

Here, both $x(t)$ and $p(t)$ take values in a Hilbert space X for $t \in [a,b]$; $F : X \times Y \times [a,b] \to X$, $G : X \times X \times [a,b] \to X$ and $\xi : X \to X$ are nonlinear operators; $N(t) : X \to Y$, another Hilbert space, is a bounded linear operator for each $t \in [a,b]$. For the family $\{A(t) : a \le t \le b\}$ of linear closed operators with adjoint operators $A^*(t)$, we assume that the domain $D(A)$ of $A(t)$ is dense in X and independent of t and that the family $\{A(t) : a \le t \le b\}$ generates a unique linear evolution system $\{U(t,s) : a \le s \le t \le b\}$ satisfying the following properties:

(a) For any $a \le s \le t \le b$, $U(t,s) \in \mathcal{L}(X)$, the space of all bounded linear operators in X, also, the mapping $(t,s) \to U(t,s)x$ is continuous for any $x \in X$;