

MULTIPEAK TRAVELING WAVE SOLUTIONS OF CAMASSA-HOLM EQUATION

Guoping Zhang

Department of Mathematics
Morgan State University, Baltimore, MD 21251, USA

Corresponding author email: guoping.zhang@morgan.edu

Abstract. In this paper we study the multipeak traveling wave solutions of the Camassa-Holm (CH) equation $m_t + m_x u + 2mu_x = 0$, $m = u - u_{xx}$. All possible smooth multipeak traveling wave solutions of the CH equation are classified and constructed explicitly.

Keywords. multipeak, traveling wave, Camassa-Holm, explicit, implicit, smooth solution, periodic solution.

AMS subject classification: 35QXX.

1 Introduction

The Camassa-Holm (CH) equation [1] is written in the following form

$$m_t + m_x u + 2mu_x = 0, \quad m = u - u_{xx}, \quad (1.1)$$

which has excited much interest in recent years. Here $u = u(x, t)$ represents the horizontal component of the fluid velocity, and $m = u - u_{xx}$ is the momentum variable. The subscripts x, t of u denote the partial derivatives of the function u w.r.t. x, t . For example, $u_t = \partial u / \partial t$, $u_{xxt} = \partial^3 u / \partial^2 x \partial t$. Similar notations will be used frequently later in this paper.

In [3], [4] and [5] Lenells gave a very thorough classification of weak solutions of CH equation by using asymptotic analysis theory. In [2] Cao, Holm and Titi found some new traveling wave solutions for a class of one-dimensional nonlinear shallow water wave equations (including CH equation) based on peakon type soliton solution. In [6] Zhang and Qiao constructed all single peak traveling wave solutions for CH equation. A few explicit solutions are given in their works. In [7] the author and his collaborators have been extensively studied CH equation.

The present paper provides an approach to construct explicit multipeak traveling wave solutions of the CH equation (1.1). In the paper I classify and construct all possible smooth multipeak traveling wave solutions of the CH equation.