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FIXED POINT THEORY AND INTEGRAL EQUATIONS

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Abstract. We consider Volterra equations of convolution type with kernels continuous on $(0, T]$, allowing for a kernel with a singularity at zero, provided it is integrable on $(0, 1)$. For such equations we may apply both Schauder's and Schaefer's theorems without having to mention the compactness or even the continuity of the mapping, since those properties will be immediately obtained from the form of the integral equation. Prototypes use kernels of the form $C(t, s) = (t - s)^{q-1}$ where $0 < q < 1$, as found throughout heat theory and in fractional differential equations of both Riemann-Liouville and Caputo type. We conclude with an example involving heat conduction.

Keywords. Integral Equations, Singular Kernels, Fixed Points, Schaefer's Theorem, Schauder's Theorem

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1 Introduction

A thesis pursued in a series of recent papers [1], [2], [3], [4] is that when Schauder's fixed point theorem is used in the study of behavior of solutions to integral equations, conditions of compactness, which are needed to invoke the theorem, need not be verified in its application, because these conditions are inherent in the nature of the equation itself. That these conditions are present by default often goes unnoticed, causing papers to be cluttered with arguments demonstrating the compactness of the mapping. Worse still is the situation in which application of fixed point theory is abandoned because of the perceived difficulties.

We will be studying nonlinear integral equations with singular kernels, but before giving the details of this problem we first recall the setting in the linear case,

$$z(t) = a(t) - \int_0^t K(t, s)z(s)ds \tag{1}$$