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AN ALGORITHMIC STABILITY TEST FOR NEUTRAL FIRST ORDER DELAY DIFFERENTIAL EQUATIONS WITH M COMMENSURATE DELAYS

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Abstract. In this paper we derive a robust algorithmic stability test to determine asymptotic stability of the zero solution of a first order neutral differential equations of the form

$$y'(t) + \alpha y'(t - m\tau) + \sum_{j=0}^{m} a_j y(t - j\tau) = 0$$

where $a_j,\ j = 0, \ldots, m$, and α and τ are constants, $|\alpha| < 1$, and $\tau > 0$. Some necessary conditions for asymptotic stability are also obtained. In proving our results, we make use of Pontryagin's theory for quasi-polynomials and Chebyshev polynomials.

Keywords. stability criteria, algorithmic stability test, commensurate delays, characteristic functions, Chebyshev polynomials.

1 Introduction

The aim of this paper is to derive a robust algorithmic stability test to determine asymptotic stability of the zero solution of the neutral differential equation of the form

$$y'(t) + \alpha y'(t - m\tau) + \sum_{j=0}^{m} a_j y(t - j\tau) = 0 \tag{1.1}$$

where a_j, $j = 0, \ldots, m$, α, and τ are constants, $|\alpha| < 1$, and $\tau > 0$. Equation (1.1) has many applications in Biology [12] and Control Theory [10]. In our previous papers [2,3], we considered first order non-neutral equations with m commensurate delays. We also studied the non-neutral equation (1.1) with $m = 1$ and complex coefficients [4]. Although there has been some study of differential equations with more than one delay (see [7,11,16, 17] and references therein), there has been no comprehensive study for several