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ANALYSIS OF TWO GRID METHOD FOR SOLVING HIGHER-ORDER FINITE ELEMENT METHODS

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Abstract. We revisit an auxiliary space preconditioning method proposed by Xu [Computing 56, 1996], in which low-order finite element spaces are employed as auxiliary spaces for solving linear algebraic systems arising from higher-order finite element discretizations of the Poisson equation. In this paper, we provide a new convergence rate estimate for this two-level method, by casting it as a standard iterative method for a semidefinite system. The convergence rate is shown to be independent of the mesh size.

Keywords. Second-order elliptic equations, finite element method, multigrid.

AMS (MOS) subject classification: 35J05, 65N30, 65N55

1 Introduction

The Poisson equation $-\Delta u = f$ and its variants arise in many applications. The geometric multigrid (GMG) method is one of the most efficient iterative methods for solving discrete Poisson or Poisson-like equations. A vast number of works have explored multigrid methods; references include the monographs and the survey papers [3, 11, 2, 23]. Though the classical multigrid algorithm based on a geometric hierarchy can be an effective solver for a well-structured grid, it is usually very difficult to obtain such a hierarchy in practice. The algebraic multigrid (AMG) method [6, 4, 21, 23, 5, 9], on the other hand, requires minimal geometric information about the underlying problem and can sometimes be employed as a "black-box" iterative solver or preconditioner for other iterative methods. The version known as the Classical AMG [6, 18] is used frequently and has been shown to be effective for a range of problems in practice. In an effort to render AMG methodologies more broadly applicable and to improve robustness, various versions have been developed; for example, see [24, 27, 15, 7].

Studies have proposed using a two-level approach to handle Poisson equations on the higher-order finite element spaces. Such an approach would consist of (1) a smoother for the Poisson equation on the higher order finite element spaces, (2) transfer operators between the higher-order finite element