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CONTROL OF STACKELBERG FOR A TWO-STROKE PROBLEM

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Abstract. In this paper we apply the notion of hierarchical control on a two-stroke problem. This notion assumes that we have two controls to determine. One which will be called Leader and the other Follower. The first control is supposed to bring the solution of the two-stroke equation subject to a finite number of constraints at rest at time $t = 0$ while the second expresses that the state does not move too far from a given state. The results are achieved by means of an observability inequality of Carleman adapted to the constraint.

Keywords. Population dynamics systems. Null controllability. Carleman inequalities. Inequality of Observability.

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1 Introduction

The concept of hierarchical control is based on the notion of Stackelberg leadership problem which is a strategic game in economics in which the leader firm moves first and then the follower firms move sequentially. The Stackelberg problem, also called Stackelberg competition, is an economic duopoly model of imperfect competition based on a non-cooperative game. It was developed in 1934 by Heinrich Von Stackelberg in his "Market Structure and Equilibrium" and represented a breaking point in the study of market structure, particularly the analysis of duopolies. Today, Stackelberg name evokes a sequential view of the game in which two firms (a Leader and a Follower) compete on the market of the same property. One is the first to act by integrating the reaction of the other company in the choices it makes in the amount it decides to put on the market. Following this notion, we want to control a two-stroke equation by acting on two controls. Let us formulate the problem more precisely. Let $N \in \mathbb{N}^*$, Ω be a bounded open subset of $\mathbb{R}^N$ with boundary Γ of class $\mathcal{C}^2$ and $y = y(t,a,x)$ be the distribution of individuals of age $a \geq 0$, at time $t \geq 0$ and location $x \in \Omega$. Let also A be the life expectancy of an individual and T a positive constant. Set $U = (0,T) \times (0,A)$, $Q = U \times \Omega$, $\Sigma = U \times \Gamma$, $Q_A = (0,A) \times \Omega$,