Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **22 (2015)** 425-440

SIMULATION OF STOCHASTIC OPTIMAL CONTROL PROBLEMS WITH SYMPLECTIC PARTITIONED RUNGE-KUTTA SCHEME

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Abstract. In this work, we obtain the symplectic partitioned Runge-Kutta (SPRK) scheme for the optimal control problem of stochastic differential equations (SDEs). In order to discretize the optimal control problem, there are two basic approaches: *discretize-then-optimize* and *optimize-then-discretize*. We mainly focus on SPRK scheme for the optimal control problem of SDEs by following the *discretize-then-optimize* approach. After we present Hamiltonian formulations for the stochastic optimal control problem, we discretize the cost functional and the state equation with the help of Runge-Kutta schemes. To obtain the optimality system, we state the discrete Lagrangian. Then, we get the stochastic adjoint pair (p_t, q_t). Our main contribution is to obtain an implicit Runge-Kutta scheme for the adjoint pair. As applications, we choose some problems from finance. We compare the numerical results with the exact solutions.

Keywords. Stochastic optimal control, Runge-Kutta discretization, symplectic partitioned Runge-Kutta scheme, Lagrange multiplier, Hamiltonian.

AMS (MOS) subject classification: 93E20, 49M15, 49M25, 65C20.

1 Introduction

SDEs have recently become increasingly important in many fields, especially in finance, economics and physics [9, 23]. Many of SDEs do not have closed form solutions. So that, numerical methods are applied to get approximate solutions of SDEs. In literature, there are many works related to numerical solutions of SDEs. In [23], Platen and Kloeden derived a deep cover of Itô-Taylor expansion which gives rise to many numerical methods yielding computation of derivatives that might be costly. Then, it is reasonable to use numerical schemes that approximate derivatives such as Runge-Kutta type schemes [4, 5, 6, 7, 23, 28, 35]. Burrage and Burrage presented a general class of stochastic Runge-Kutta methods in [4]. In [34], Tian and Burrage discussed two-stage diagonally implicit stochastic Runge-Kutta methods with