Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **22 (2015)** 481-488
 http://www.watam.org

AN EXISTENCE RESULT FOR A NEUMANN PROBLEM

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Abstract. The main result of this paper deals with the existence of at least one positive solution for a second order Neumann boundary value problem. Such a result is obtained by using an abstract coincidence point theorem that allows to get our conclusion under non standard conditions on the nonlinearity.

Keywords. second-order Neumann problem, existence result, coincidence point, positive solution, fixed point.

AMS (MOS) subject classification: 34B15, 34B18.

1 Introduction

In the present note we study the existence of at least one positive solution for the following one dimensional quasi-linear Neumann boundary value problem

$$\begin{cases} -u'' + M(t)u = f(t,u,u') & \text{in } [0,1] \\ u'(0) = u'(1) = 0, \end{cases} \tag{1}$$

where $M : [0,1] \to \mathbb{R}$ is a continuous and positive function, while $f : [0,1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a continuous function with $f(t,0,0) > 0$, for every $t \in [0,1]$.

There is a large literature concerning existence results for a two-point Neumann boundary value problem obtained adopting different type of approaches: monotone methods [6], upper and lower solutions [7], topological transversality theorem [10, 4], topological degree [13], fixed point theorems as well as fixed point index theory on a cone [14].

More recently, some authors have paid attention to the existence and the multiplicity of positive solutions for problem (1) provided that the nonlinearity does not depend on the derivative of the solution, i.e for the problem

$$\begin{cases} -u'' + Mu = g(t,u) & \text{in } [0,1] \\ u'(0) = u'(1) = 0, \end{cases} \tag{2}$$

where $g : [0,1] \times \mathbb{R} \to \mathbb{R}$ is a suitable continuous function and M is a positive constant. In particular, the existence of at least one solution has

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