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LIAPUNOV FUNCTIONALS AND STABILITY IN FRACTIONAL DIFFERENTIAL EQUATIONS

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Dedicated to Professor Theodore A. Burton on the occasion of his 80th birthday

Abstract. This project is devoted to developing Liapunov direct method for fractional differential equations. The method consists of constructing a system related scalar function which enables investigators to analyze the qualitative behavior of solutions of a differential equation without actually finding its solutions. We first convert a class of fractional differential equations to integral equations with singular kernels and then construct Liapunov functionals for the integral equations to deduce conditions on boundedness, stability, and L^p-solutions. It has long been our view that, since the fractional differential equation can be written as an integral equation with a completely monotone kernel, it is possible to construct a Liapunov functional that is of positive type. This is another installment supporting that belief.

Keywords. Liapunov functionals, asymptotic stability, fractional differential equations

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1 Introduction

We study the stability properties of a fractional differential equation of Caputo type

$$ {}^cD^q x = -ax(t) + G(t,x), \quad x(0) = x_0, \quad 0 < q < 1, \tag{1.1}$$

with $a > 0$ and $G : [0,\infty) \times \Re \to \Re$ being continuous. Under this continuity condition, it has been shown that the initial value problem (1.1) can be inverted to the nonlinear Volterra integral equation of the second kind

$$x(t) = x(0) + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} \left[-ax(s) + G(s,x(s))\right] ds \tag{1.2}$$

where Γ is the gamma function (Diethelm and Ford [14], Lakshmikantham et al [17, p.54]). We observe that (1.2) is a singular integral equation with a convex kernel. If (1.1) is written in a special form

$$ {}^cD^q x = ax(t) + g(t,x(t)), \quad x(0) = x_0, \quad 0 < q < 1, \tag{1.3}$$

where a is a constant, then the solution of (1.3) is given by

$$x(t) = \Phi_0(t)x_0 + \int_0^t \Phi(t-s)g(s,x(s))ds, \tag{1.4}$$