

A NEW CLASS OF HIGHER-ORDER NONLINEAR BOUNDARY VALUE PROBLEMS WITH NONLOCAL STRIP-CONDITIONS

Ravi P. Agarwal^{1,2}, Bashir Ahmad², Ahmed Alsaedi² and Nada Al-Malki²

¹Department of Mathematics, Texas A&M University, Kingsville, TX 78363-8202, USA

²Department of Mathematics, Faculty of Science, King Abdulaziz University,
P.O. Box 80203, Jeddah 21589, Saudi Arabia

E-mail: Agarwal@tamuk.edu (RPA), bashirahmad_qau@yahoo.com (BA),
aalsaedi@hotmail.com (AA), mass_nana@hotmail.com (NA)

Abstract. We investigate a new class of boundary value problems of nonlinear n th-order ordinary differential equations and inclusions with nonlocal strip conditions. We consider two strip conditions such that the contribution due to a strip occupying the position $(0, \xi)$ is related to the value of the unknown function at the left-end point of the given interval while the contribution due to a strip placed at the position $(\zeta, 1)$ is connected to the linear combination of values of the unknown function and its derivative at an interior nonlocal position η in the segment of the given interval off the two strips, where $0 < \xi < \eta < \zeta < 1$. The given single valued and multivalued problems are solved by means of appropriate tools of the fixed point theory and sufficient criteria for the existence of solutions for the problems at hand are presented. We explain the main results with the aid of examples. We also extend our discussion to some companion problems.

Keywords. Differential equations and inclusions; n th-order; integral boundary conditions; nonlocal; fixed point

AMS (MOS) subject classification: 34B10, 34B15, 34A60.

1 Introduction

The subject of nonlocal nonlinear boundary value problems has evolved into an interesting field of research owing to its extensive applications in diverse disciplines such as fluid mechanics, geophysics, mathematical physics, etc. It has been observed that some peculiarities of physical, chemical or other processes occurring at some intermediate positions of the given domain cannot be expressed in terms of classical boundary conditions. To describe this situation, nonlocal conditions were introduced in [1]. Such conditions relate the values of the unknown function at the boundary points with its values at some interior positions of the domain. On the other hand, investigation of nonlinear boundary value problems helps to examine the effect of nonlinearity on the solutions of the given problem. For details and development