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PARAMETER PROPERTIES OF THE RESOLVENT OF THE OPERATOR OF ABSTRACT CAUCHY PROBLEM

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Abstract. This work investigates the continuity and differentiability with respect to parameters of the resolvents of the operators of abstract Cauchy problems, and studies the case that the domain of the operators depend on the parameters. The obtained results are effectively and conveniently applied to an initial-boundary value problem of partial differential equation.

Keywords. abstract Cauchy problem, resolvent of operator, continuity, differentiability, parameter

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1 Introduction

Consider the abstract Cauchy problem

$$\begin{aligned} u'(t) &= A(\varepsilon)u(t) + f(t, u(t), \varepsilon), \qquad \text{for } t \geq 0, \\ u(0) &= u_0, \end{aligned} \tag{1.1}$$

where $A(\varepsilon)$ is a closed linear operator on a Banach space and ε is the parameter.

A typical way to study the solutions of the abstract Cauchy problem is to employ C_0-semigroup theory for densely defined operators $A(\varepsilon)$ and integrated semgroup theory for non-densely defined operators $A(\varepsilon)$. According to the C_0-semigroup theory and integrated semigroup theory, the solutions of the abstract Cauchy problem are mainly expressed in terms of C_0-semigroup $T(t, \varepsilon)$ or integrated semigroup $S(t, \varepsilon)$ as well as the resolvent $(\lambda I - A(\varepsilon))^{-1}$ of the operator $A(\varepsilon)$. Furthermore, a number of results (see [1] and references therein] show that the semigroups can be mainly expressed and in terms of the resolvant $(\lambda I - A(\varepsilon))^{-1}$ of the operator $A(\varepsilon)$. Thus, determining the parameter properties of the resolvent $(\lambda I - A(\varepsilon))^{-1}$ is an effective way to obtain the same properties for the semigroups, which are used to obtain the same properties of solutions of the abstract Cauchy problems.

Much research work has devoted in studying the continuity in parameters and differentiability with respect to parameters of the resolvent $(\lambda I - A(\varepsilon))^{-1}$