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INFINITESIMAL STRETCHING AND FOLDING I

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Abstract. In [1] it is noted that stretching and folding in nonlinear ODEs, unlike discrete maps such as the Hénon map or twist and flip map, appears "infinitesimally". In [2] it is noted that solutions of nonlinear ODEs can be decomposed into an infinite product of "infinitesimal compositions" which suggests that it may be possible to approximate solutions of ODEs in a new way that is a combination of numerical integration and finite difference methods. In this paper, I will show that such a decomposition is possible and that the known complex dynamics that occurs in second and third order ODEs can be represented as what will be called *iterated infinitesimal stretching and folding (ISF) maps*. It will be seen that ISF maps share some properties common to iterated maps and also share some properties with numerical solutions of ODEs. In this sense, ISF maps are a hybrid map that makes use of the best features of iterated maps and numerical integration algorithms while retaining the inherent dynamics of interest in an ODE. It will also be suggested that this approach will apply equally well to classical physics and to the dynamics of social organizations.

Keywords. Chaos, psychology, neuroscience, numerical integration, sociology, physical science, complexity, dynamical synthesis.

AMS (MOS) subject classification: 37D45.

1 Introduction

In [1] it is noted that stretching and folding in nonlinear ODEs, unlike discrete maps such as the Hénon map or twist and flip map, appears "infinitesimally". In [2] it is noted that the solution of nonlinear ODE's can be decomposed into an infinite product of compositions. In this paper, I will show that the known complex dynamics that occurs in second and third order ODEs can be represented as iterated infinitesimal stretching and folding (ISF) maps. ISF maps share some properties common to iterated maps and also share some properties with numerical solutions of ODEs. In this sense, ISF maps are a hybrid map that makes use of the best features of iterated maps and numerical integration algorithms.

It is clear that within the laws of physics, complexity can be explained by the combination of stretching and folding, see [3]. In [4] we showed how this occurs by combing a weighted average of a Bernoulli map (stretching) with an almost periodic map (folding). In [1] I showed how stretching and folding could be recognized