Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **22 (2015)** 395-407

http://www.watam.org

EXISTENCE OF SOLUTIONS VIA VARIATIONAL METHODS FOR A PROBLEM WITH NONLINEAR BOUNDARY CONDITIONS ON THE HALF-LINE

Ouidad Frites,[1] Toufik Moussaoui[1] and Donal O'Regan[2]

[1]Laboratory of Fixed Point Theory and Applications
École Normale Supérieure, Kouba, Algiers. Algeria
ofrites@yahoo.fr, moussaoui@ens-kouba.dz
[2]School of Mathematics, Statistics and Applied Mathematics
National University of Ireland
Galway, Ireland
NAAM Research Group, King Abdulaziz University
Jeddah, Saudi Arabia
donal.oregan@nuigalway.ie

Abstract. The aim of this paper is to study the existence of solutions for a boundary value problem posed on the half-line with nonlinear boundary conditions using nonsmooth critical point theory.

Keywords. Critical points, Minimization, Szulkin-type functionals, Ricceri's critical point theorem.

AMS (MOS) subject classification: 47J20, 49J40.

1 Introduction

For a function $j : \mathbb{R} \longrightarrow \mathbb{R} \cup \{+\infty\}$ which is proper ($i.e., D(j) = \{z \in \mathbb{R} : j(z) < +\infty\} \neq \emptyset$), convex and continuous with $j(0) = 0$ we consider the problem, denoted by (P_λ):

$$\begin{cases} -u'' + u = q_1(t)f(t, u(t)) + \lambda q_2(t)g(t, u(t)), \quad a.e. \quad t \in (0, +\infty), \\ u'(0) \in \partial j(u(0)), \\ u(\infty) = 0, \end{cases}$$

where $\lambda > 0$, $q_1, q_2 \in L^1(\mathbb{R}^+)$ and $f, g : [0, +\infty) \times \mathbb{R} \longrightarrow \mathbb{R}$ are continuous functions satisfying the conditions:
(H_f) $|F(t, x)| \leq H_1(x)$, for all $t \in (0, \infty)$, $x \in \mathbb{R}$;
(H_g) $|G(t, x)| \leq H_2(x)$, for all $t \in (0, \infty)$, $x \in \mathbb{R}$;
where $H_1, H_2 : \mathbb{R} \longrightarrow \mathbb{R}$ are continuous functions such that for any constant $R > 0$,

$$\sup\{|H_i(\frac{1}{p(t)}y)| : t \in [0, +\infty), y \in [-R, R]\} < \infty, \, i = 1, 2.$$