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BIFURCATION OF LIMIT CYCLES FROM SOME UNIFORM ISOCHRONOUS CENTERS

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Abstract. This article concerns with the weak 16–th Hilbert problem. More precisely, we consider the uniform isochronous centers

$$\dot{x} = -y + x^{n-1}y, \qquad \dot{y} = x + x^{n-2}y^2,$$

for $n = 2, 3, 4$, and we perturb them by all homogeneous polynomial of degree $2, 3, 4$, respectively. Using averaging theory of first order we prove that the maximum number $N(n)$ of limit cycles that can bifurcate from the periodic orbits of the centers for $n = 2, 3$, under the mentioned perturbations, is 2. We prove that $N(4) \geq 2$, but there is numerical evidence that $N(4) = 2$. Finally we conjecture that using averaging theory of first order $N(n) = 2$ for all $n > 1$. Some computations have been made with the help of an algebraic manipulator as mathematica.

Keywords. Periodic solution, uniform isochronous centers, averaging theory, weak Hilbert problem.

AMS (MOS) subject classification: 37G15, 37C80, 37C30.

1 Introduction and statement of the main results

The second part of the 16th Hilbert's problem asks for the maximum number $H(n)$ and position of limit cycles for all planar polynomial differential systems of degree n, for more details on the 16th Hilbert's problem see [8, 10, 11], and the references quoted therein. The problem on the number $H(n)$ remains open, even for $n = 2$, and a general result about the configurations of limit cycles in planar polynomial differential systems can be found in [13].

A weak form of the 16th Hilbert's problem, known now as the *weak 16th Hilbert's problem* was proposed by Arnold [1], asking for the maximum number $Z(m, n)$ of isolated zeros of Abelian integrals of all polynomial 1–form of degree n over algebraic ovals of degree m, for more information on the weak 16th Hilbert's problem see [5, 9, 18], and the hundreds of references quoted