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EXISTENCE OF PERIODIC SOLUTIONS OF TOTALLY NONLINEAR NEUTRAL DYNAMIC EQUATIONS WITH FUNCTIONAL DELAY ON A TIME SCALE

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Abstract. Let $\mathbb{T}$ be a periodic time scale. Using a modification of Krasnoselskii's fixed point theorem due to Burton, we show that the totally nonlinear dynamic equation with functional delay

$$x^{\triangle}(t) = -a(t)\,h(x^{\sigma}(t)) + c(t)Q^{\widetilde{\triangle}}(x(t-r(t))) + G(t, x(t), x(t-r(t))),$$

where $t \in \mathbb{T}$, $f^{\triangle}$ is the $\triangle$-derivative on $\mathbb{T}$ and $f^{\widetilde{\triangle}}$ is the $\triangle$-derivative on $(id - r)(\mathbb{T})$, has a periodic solution. We invert this equation to construct a sum of a compact map and a large contraction which is suitable for applying the Burton-Krasnoselskii's theorem. The results obtained here extend the results in the literature.

Keywords. Fixed point, large contraction, periodic solutions, time scales, nonlinear neutral dynamic equations.

AMS (MOS) subject classification: Primary 34K13, 34A34; Secondary 34K30, 34L30.

1 Introduction

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. The study of dynamic equations on time scales is a fairly new subject, and research in this area is rapidly growing (see [1, 3, 4, 5, 6, 8, 9, 10, 11, 16, 17, 18], and the papers therein). The theory of dynamic equations unifies the theories of differential equations and difference equations. We suppose that the reader is familiar with the basic concepts concerning the calculus on time scales for dynamic equations. Otherwise one can find in Bohner and Peterson books [10, 11] most of the material needed to read this paper. We start by giving some definitions necessary for our work. The notion of periodic time scales is introduced in Atici et al. [9] and Kaufmann and Raffoul [17]. The following two definitions are borrowed from [9, 17].