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FRACTIONAL DIFFERENTIAL EQUATIONS, TRANSFORMATIONS, AND FIXED POINTS

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Abstract. This is the continuation of an earlier study of a scalar fractional differential equation of Riemann-Liouville type

$$D^q x(t) = -f(t, x(t)), \quad \lim_{t\to 0^+} t^{1-q}x(t) = x^0 \in \Re, 0 < q < 1, \tag{a}$$

in which we first invert it as a Volterra integral equation

$$x(t) = x^0 t^{q-1} - \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} f(s, x(s))ds \tag{b}$$

and then transform it into

$$y(t) = F(t) + \int_0^t R(t-s)\left[y(s) - \frac{f(s+T, y(s))}{J}\right]ds \tag{c}$$

where $y(t) = x(t+T)$. Here, F is uniformly continuous, converges to zero, and is in $L^1[0,\infty)$, R is completely monotone with $\int_0^\infty R(s)ds = 1$, and J is an arbitrary positive constant.

Our work here consists of extracting qualitative properties of solutions of (c). First we obtain results showing solutions are either bounded, asymptotically constant, asymptotically periodic, or in $L^p[0,\infty)$. Next, we show that for (c) Schauder's fixed point theorem can be reduced to a simple parallel of Brouwer's theorem. An example is given showing how to construct self-mapping sets. One of the main points is that we deal with problems having large growth properties, such as $f(t,x) = x^{2n+1}$ with n an arbitrary positive integer. Admissible values of q are related to the growth of f.

Keywords. fractional differential equations, Riemann-Liouville operators, singular kernels, integral equations, fixed points

AMS (MOS) subject classification: 34A08, 34A12, 45D05, 45G05, 47H09, 47H10

1 Introduction

There are wide classes of integral equations which can be mapped by a simple transformation (derived in Section 6 of [5]) into a standard form and which is so fixed point friendly that Schauder's fixed point theorem applied to the natural mapping defined by that standard form is almost as simple as Brouwer's fixed point theorem. These classes include fractional differential equations of Caputo type, many classical integral equations of type displayed in [5, p. 5650], and fractional differential equations of Riemann-Liouville type treated in [2]. There