Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **22 (2015)** 293-324

TWO-SCALE ASYMPTOTIC ANALYSIS OF SINGULARLY PERTURBED ELLIPTIC DIFFERENTIAL EQUATIONS ON LARGE PERIODIC NETWORKS

Erik Kropat[1] and Silja Meyer-Nieberg[2] and Gerhard-Wilhelm Weber[3*]

[1]Institute for Applied Computer Science
University of the Bundeswehr Munich, Werner-Heisenberg-Weg 39, 85577 Neubiberg, Germany

[2]Institute for Theoretical Computer Science, Mathematics and Operations Research
University of the Bundeswehr Munich, Werner-Heisenberg-Weg 39, 85577 Neubiberg, Germany

[3]Institute of Applied Mathematics
Middle East Technical University, 06531 Ankara, Turkey

Abstract. Differential equations on large periodic networks cover a wide range of phenomena in multi-scale physics and engineering sciences. Typically, the effective behaviour of these physical systems is governed by the geometry and the intrinsic effects of the underlying periodic microstructure. Generally, even numerical solutions can not be obtained in a reasonable time because of the complexity and density of the network, the high number of branches and nodes as well as the highly oscillating coefficients. We propose a two-scale asymptotic method for elliptic differential equations on one-dimensional periodic manifolds. The limit analysis of the singularly perturbed system leads to an easy-to-solve approximating model characterizing the effective behaviour of the system. The coefficients of these macroscopic models reflect the geometry of the underlying periodic microstructure and provide a full characterization of the network model on a global scale. Error estimations confirm the quality of the macroscopic models obtained by the asymptotic method. Finally, we demonstrate the effectiveness of the two-scale method at the example of diffusion-advection-reaction equations on one-dimensional manifolds.

Keywords. Asymptotic analysis, two-scale method, networks, diffusion-advection-reaction equations, boundary value problems on graphs and networks

AMS (MOS) 34B45, 34E13, 34E05, 34E10

1 Introduction

Differential equations on very large networks with a *periodic microstructure* are of considerable importance in many applications ranging from engineering sciences to multi-scale physics. The underlying graph-structures are often

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*Honorary positions: Faculty of Economics, Business and Law, University of Siegen, Germany; Center for Research on Optimization and Control, University of Aveiro, Portugal; Faculty of Science, Universiti Teknologi Malaysia (UTM), Skudai, Malaysia.