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APPROXIMATE SOLUTIONS OF SOME CLASSES OF FUNCTIONAL EQUATIONS BY A MODIFIED NEUMANN SERIES

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Abstract. A modified Neumann series is used to approximate the analytical solutions of some classes of functional equations. The Neumann series may converge slowly to the analytical solution or might even be divergent. Nonlinear transformations are implemented to speed up the convergence or to evaluate the summation if the series is divergent. Four practical examples are solved to demonstrate the applicability of this approach. A theorem on convergence of Neumann series for a general class of nonlinear Abel-Volterra equations is proved.

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1 Introduction

Some boundary value problems lead to the Fredholm-type integral equations. There is also a strong connection between ordinary differential equations and the Volterra-type integral equations, see [1]. Integral equations are normally easier to solve than ODEs and PDEs by numerical means. Some classes of integral equations are:
Nonlinear Fredholm integral equations of the second kind:

$$x(t) = g(t) + \lambda \int_a^b K(t,s,x(s))ds, \;\; t \in [a,b], g \in C[a,b] \;\text{ and }\; K \in C\big([a,b]^2 \times \mathbb{R}\big). \tag{1}$$

The nonlinear Abel-Volterra integral operators of the second kind are represented by:

$$x(t) = g(t) + \int_0^t \frac{K(t,s,x(s))}{(t-s)^\alpha} ds, \;\; 0 \leqslant \alpha < 1, \;\; t \in [0,b]. \tag{2}$$

In passing we note that the ordinary differential equations:

$$x'(t) = K(t,x(t)), \;\; \text{with } \; x(0) = c, \tag{3}$$

is equivalent to

$$x(t) = c + \int_0^t K(s,x(s))ds. \tag{4}$$