STABILITY AND SQUARE INTEGRABILITY OF SOLUTIONS OF NONLINEAR THIRD ORDER DIFFERENTIAL EQUATIONS

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Abstract. The aim of this paper is to study the asymptotic properties and square integrability of solution and their derivatives to nonlinear differential equations of the third order. The authors give sufficient conditions for such behavior. The main results are illustrated with an example.

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This paper is dedicated to Ted Burton on the occasion of his eightieth birthday.

1   Introduction

In the present paper, we investigate some asymptotic properties of solutions of the third order nonlinear differential equation

$$[P(x(t))x'(t)]'' + a(t)(Q(x(t))x'(t))' + b(t)(R(x(t))x'(t)) + c(t)f(x(t)) = 0, \tag{1}$$

where the functions a, b, $c : [0, \infty) \to [0, \infty)$, P, Q, $R : \mathbb{R} \to [0, \infty)$, and $f : \mathbb{R} \to \mathbb{R}$ are continuous, and $xf(x) > 0$ for $x \neq 0$.

   Determining the asymptotic stability and square integrability of solutions to second order differential equations has been a very active area of research over the years; see, for example, the monograph of Bartušek, Z. Došlá, and Graef [2], as well as the papers [5, 9, 10, 11] and the references contained within. By comparison, the study of these properties for third order equations has received considerably less attention in the literature. For some early well