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ASYMPTOTICALLY STABLE SOLUTIONS OF A SYSTEM OF NONLINEAR DIFFERENTIAL EQUATIONS

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Abstract. The existence of asymptotically stable solutions of the following system of nonlinear differential equations

$$x'(t) + A(t)x(t) = f(t, x(t)), \;\; x(0) = x_0, \;\; t \geq 0,$$

is studied in this paper. Schauder's fixed point theorem is used in the analysis. The stability that is studied in this paper is not the standard Liapunov stability, which is commonly studied by the researchers in the field of differential equations..

Keywords. Asymptotically stable solution, nonlinear integral equation, Schauder fixed point theorem.

AMS (MOS) subject classification: 34D05, 45D05.

1 Introduction

Let n be a positive integer. Consider following initial value problem (I.V.P.):

$$x'(t) + A(t)x(t) = f(t, x(t)), \;\; x(0) = x_0, \;\; t \geq 0, \tag{1}$$

where $x = (x_1, x_2, ..., x_n)^T \in R^n, x_0 = (x_0^1, x_0^2, ..., x_0^n)^T \in R^n, f = (f_1, f_2, ..., f_n)^T$, and A is an $n \times n$ diagonal matrix with diagonal elements $a_i, i = 1, 2, ...n$.

The basic assumptions throughout the paper are that for each $i = 1, 2, ..., n$, $f_i : R_+ \times R^n \to R$ is continuous, and $a_i : R_+ \to R$ is continuous, where $R_+ = [0, \infty)$.

Our objective is to show the existence of asymptotically stable solutions of the I.V.P. (1) in terms of the following definition found in [1], [2], and [6].

Definition 1. A function x is said to be asymptotically stable solution of the I.V.P. (1) if for every $\epsilon > 0$, there exists a $T = T(\epsilon)$ such that for every $t \geq T$, and for every other solution y of (1), $|x(t) - y(t)| \leq \epsilon$.

We assume that the function f does not satisfy a Lipschitz condition in x on a domain $D \subset R_+ \times R^n$ containing the initial point $(0, x_0)$. This will allow the I.V.P. (1) to have more than one solution. As indicated in [6] that it is crucial to have the non-uniqueness of solutions for the kind of stability property that we study in this paper. At the end of the paper, in Corollary