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HOPF BIFURCATION ANALYSIS IN DELAYED DIFFERENTIAL EQUATIONS WITH TWO DELAYS

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Abstract. In this manuscript, we provide a framework for Hopf singularity for a system of two general delayed differential equations (DDEs) with two different delays. The distribution of the eigenvalues for the linearized system at an equilibrium point is studied in detail. Explicit conditions for the system to undergo a Hopf bifurcation are established and the corresponding normal forms up to the third order terms are derived. A mathematical model describing tumor and immune system interaction is presented to demonstrate our theoretical results.

Keywords. Delay differential equations, Hopf bifurcation, normal form, tumor-immune response system.

AMS (MOS) subject classification: 34K18

1 Introduction

In order to study the dynamics of tumor-immune system interaction, Bi and Ruan [1] studied the following general two-dimensional delay differential equation system with two delays,

$$\begin{cases} x'(t) = f_1(x(t-\tau), x(t), y(t)), \\ y'(t) = f_2(x(t-\rho), x(t), y(t)), \end{cases} \tag{1}$$

where x, y are functions of t, $f_1(x, x, y), f_2(x, x, y) \in C^r(\mathbb{R}^3, \mathbb{R})$, $r \geq 3$ with $f_1(0, x, y) = 0, f_2(x, x, 0) \geq 0$. For delays τ and ρ satisfying $\tau = \rho$, they provided a detailed analysis for the stability of equilibria, the existence of Hopf bifurcation, Bautin bifurcation, and Hopf-Hopf bifurcation along with the calculations for the normal forms. For delays $\tau \neq \rho$, they performed the analysis by assuming that one delay is equal to zero or fixed, so the analysis was carried out essentially under one delay assumption. In this paper, we will study Sys.(1) but only under the condition of $f_1, f_2 \in C^r(\mathbb{R}^3, \mathbb{R})$, $r \geq 3$. We will use a different approach developed in [8,9] to perform a detailed analysis for Sys.(1) for $\tau \neq \rho$. In particular, assuming that $\tau, \rho \geq 0$, we will carry out a detailed discussion for the distribution of the roots of the characteristic