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ON KANTOROVICH AND ARITHMETIC-HARMONIC MEAN INEQUALITIES

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Abstract. Arithmetic-harmonic mean inequality and Kantorovich inequality has been generalized for both scalar and matrix cases . Exact expression has been obtained for the difference between arithmetic and harmonic means .

Keywords. Kantorovich inequality; Arithmetic mean; Harmonic mean ; Parallel sum ; Positive definite matrix ; generalized inverse

AMS (MOS) subject classification: This is optional. But please supply them whenever possible.

1 Introduction

Starting from Bellman's inequality and it's generalizations (discrete and nonlinear) , one discovers the relevant relation between inequalities and their applications in dynamical systems .

In fact , Inequalities play a vital role in all fields of mathematics , and they are very strong tools for investigating approximately all problems of science including the investigation of dynamical systems behavior (local and global) , and stability problems . Examples of such applications can be found in [[1] , [9] , [12] , [10] , [13]].

The following inequality

$$\left(\sum_{i=1}^{n} \frac{\alpha_i}{x_i}\right)^{-1} \leq \sum_{i=1}^{n} \alpha_i x_i \quad , \tag{1}$$

and it's reversal

$$\sum_{i=1}^{n} \alpha_i x_i \leq \frac{(H+h)^2}{4Hh} \left(\sum_{i=1}^{n} \frac{\alpha_i}{x_i}\right)^{-1} \quad , \tag{2}$$

$0 < h \leq x_i \leq H$, $\alpha_i > 0, \quad 1 \leq i \leq n, \sum \alpha_i = 1$ are well-known . It is interesting to notice that inequality (1) is called the arithmetic-harmonic mean inequality, while (2) has another name : Kantorovich inequality . This assures that each of inequalities (1) and (2) has it's own importance . These