Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **22 (2015)** 255-262

STRETCHING AND FOLDING TRANSITIONS IN THE HÉNON INFINITESIMAL DIFFEOMORPHISM

Ray Brown

The EEASI Corporation
Houston, Texas, 77057 USA

Abstract

In [1] It was noted that, in the Smale horseshoe, the degree of stretching and folding was fixed. However, as also noted in [2,3,4] there is no reason to require stretching and folding to be only have a fixed value. In this is paper, we will explore how the dynamics of the Hénon ID vary with the duration of stretching and folding by varying the ”step size” parameter in the ID of the Hénon map over a range of 0.00001 to 1. Additionally, the Hénon ID provides an entirely different view of the transition to chaos from the traditional period doubling route. Through the increase in the duration of stretching and folding, what the analysis shows is, that in place of period doubling, catastrophic transitions occur that may more accurately represent what we see in nature.

Keywords. Chaos, natural science, complexity, dynamical synthesis.

AMS (MOS) subject classification: 37D45.

1 Introduction

There is little doubt that stretching and folding is the force behind complex dynamics [1]. However, in contrast to its first use in proving a very difficult mathematical theorem, stretching and folding in the natural world need not be uniform [2,3,4]. It may occur in very small increments such as occurs in physics or in very large increments such as occurs in weather. The amount or duration of stretching and folding need not be equal either. This is also illustrated by weather. In general, by recognizing that the duration and combination of stretching and folding drives complexity, it may be possible to improve our ability to predict complex dynamics generally.

In this paper we examine the Hénon ID in which stretching and folding will be equal and represented by the ID parameter h. Unequal stretching and folding will be treated in a later paper.

The point of this paper is to introduce the concept of using stretching and folding in varying durations to obtain a wider range of dynamics and to understand how dynamics can evolve as stretching and folding varies or evolves. This is particularly important for natural (weather, geology, biology, etc) systems and human