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THE THEORY OF INFINITESIMAL DIFFEOMORPHISMS: AN INTRODUCTION

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Abstract. Biological and social sciences are hard pressed to apply the laws of Newton due to the multiplicity of parameters affecting the dynamics of such systems; hence the reliance on statistical methods. However, statistical methods do not predict cause-effect relationships. Rather, statistics provides correlations between dynamics of populations. There is a need for a "calculus" that can be used to predict causal relationships in biological and social systems that is based on the operative factors of these systems: complexity.

Complexity has been extensively analyzed by mathematicians and definitively explained by Smale [1,2] through the use of the horseshoe paradigm. The essence of this paradigm is that complexity arises, in its simplest form, from the operation of two dynamics: Stretching and folding. By devising a "calculus" based on stretching and folding we may be better positioned to predict causal relationships in the biological and social sciences. The value of a calculus of stretching and folding to biological and social systems to derive dynamical systems has been presented in [2,3,4,5,6,7]. The calculus of stretching and folding is mathematically formulated in the concept of an infinitesimal diffeomorphism (ID), [4]. In this paper I present a survey of the presently known properties of IDs.

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1 Introduction: How does the study of IDs assist and advance the work of other scientific enterprises?

Infinitesimal diffeomorphisms (IDs) [4] are transformations on a manifold that can closely approximate the solution of a differential equation. However, they are a legitimate subject of analysis in their own right due to (1) their potential application in the biological and social sciences as seen in [7]; (2) their use in the numerical approximation of the solutions of ODEs; (3) their use as closed form diffeomorphisms having complex dynamics that are