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SYMMETRIC RELATIVE EQUILIBRIA IN THE FOUR-VORTEX PROBLEM WITH THREE EQUAL VORTICITIES

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Abstract. We examine in detail the relative equilibria in the 4-vortex problem when three vortices have equal strength, that is, $\Gamma_1 = \Gamma_2 = \Gamma_3 = 1$ and Γ_4 is a real parameter. We give the exact number of relativa equilibria and bifurcation values. We also study the relative equilibria in the vortex rhombus problem.

Keywords. Four-vortex problem, collinear relative equilibria, concave relative equilibria, convex relative equilibria, Gröbner bases.

AMS (MOS) subject classification: 76F20, 74F10.

1 Introduction

The origins of vortex dynamics lie in the famous work of Helmholtz of 1858, where he introduced the concepts of vortex line, vortex filament and derived the vorticity equation for an ideal incompressible fluid [8]. Helmholtz also introduced planar point vortices and their equations of motion in order to model a 2-dimensional slice of columnar vortex filaments. Some years later, Kirchhoff (1876) gave a Hamiltonian formulation of Helmholtz's equations for point vortices (see [10] for more details). This model has been widely used to provide a finite-dimensional approximation to the evolution of vorticity in fluid dynamics. Kirchhoff proved that n point vortices in the plane, with positions $z_i = (x_i, y_i) \in \mathbb{R}^2$ and vortex strength $\Gamma_i \neq 0 \in \mathbb{R}$ for $i = 1, \dots, n$, satisfy the equations

$$\Gamma_i \frac{dx_i}{dt} = \frac{\partial H}{\partial y_i}, \qquad \Gamma_i \frac{dy_i}{dt} = -\frac{\partial H}{\partial x_i}, \tag{1}$$

where

$$H = -\frac{1}{2} \sum_{i<j} \Gamma_i \Gamma_j \log[(x_i - x_j)^2 + (y_i - y_j)^2].$$