Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **22 (2015)** 177-188
 http://www.watam.org

EXISTENCE AND CONVERGENCE THEOREMS FOR λ-HYBRID SET-VALUED MAPPINGS IN HILBERT SPACES

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Abstract. In this paper, we define a new class of set-valued mappings in a Hilbert space, called λ-*hybrid*. We prove some fixed point theorems for mappings in this class. Finally, we establish weak convergence of modified SP-iteration to a fixed point of λ-hybrid set-valued mappings in Hilbert spaces. Our results extend many known recent results in the literature.

Keywords. Fixed point, Hilbert spaces, SP-iteration, λ-hybrid set-valued mapping, weak convergence.

AMS (MOS) subject classification: 47H09, 47H10.

1 Introduction

Let H be a real Hilbert space and D be a nonempty subset of H. Let $CB(D)$ and $K(D)$ denote the families of nonempty closed bounded subsets and nonempty compact subsets of D, respectively. The *Hausdorff metric* on $CB(D)$ is defined by

$$H(A,B) = \max\left\{\sup_{x\in A}\operatorname{dist}(x,B), \sup_{y\in B}\operatorname{dist}(y,A)\right\} \quad \text{for } A,B \in CB(D),$$

where $\operatorname{dist}(x,D) = \inf\{\|x-y\| : y \in D\}$. A single-valued mapping $T : D \to D$ is called *nonexpansive* if $\|Tx - Ty\| \leq \|x-y\|$ for all $x, y \in D$. A set-valued mapping $T : D \to CB(D)$ is called *nonexpansive* if $H(Tx,Ty) \leq \|x-y\|$ for all $x, y \in D$. An element $z \in D$ is called a *fixed point* of $T : D \to D$ (respectively, $T : D \to CB(D)$) if $z = Tz$ (respectively, $z \in Tz$). The set of fixed points of T is denoted by $F(T)$. A set-valued mapping $T : D \to CB(D)$ is called *quasi-nonexpansive* if $F(T) \neq \emptyset$ and $H(Tx,Tz) \leq \|x-z\|$ for all