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ON STUDY OF THE ASYMPTOTIC BEHAVIOR OF SOME RATIONAL DIFFERENCE EQUATIONS

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Abstract. In this article, we study the periodicity, the boundedness and the global stability of the positive solutions of the following nonlinear difference equation

$$x_{n+1} = ax_n + \frac{bx_{n-1} + cx_{n-2} + fx_{n-3} + rx_{n-4}}{dx_{n-1} + ex_{n-2} + gx_{n-3} + sx_{n-4}}\ , \qquad n = 0, 1, 2, ...$$

where the coefficients $a, b, c, d, e, f, g, r, s \in (0, \infty)$, while the initial conditions x_{-4},x_{-3},x_{-2}, x_{-1}, x_0 are arbitrary positive real numbers. Some numerical examples will be given to illustrate our results.

Keywords. Difference equations, prime period two solution, boundedness character, locally asymptotically stable, global attractor, global stability.

AMS (MOS) subject classification: 39A10,39A11,39A99,34C99.

1 Introduction

The qualitative study of difference equations is a fertile research area and increasingly attracts many mathematicians. This topic draws its importance from the fact that many real life phenomena are modeled using difference equations. Examples from economy, biology, etc. can be found in [2, 3, 20, 23, 33]. It is known that nonlinear difference equations are capable of producing a complicated behavior regardless its order. This can be easily seen from the family $x_{n+1} = g_\mu (x_n)$, $\mu > 0$, $n \geq 0$. This behavior is ranging according to the value of μ, from the existence of a bounded number of periodic solutions to chaos.

There has been a great interest in studying the global attractivity, the boundedness character and the periodicity nature of nonlinear difference equations. For example, in the articles [1, 9–19, 24–34] closely related global convergence results were obtained which can be applied to nonlinear difference equations in proving that every solution of these equations converges to a period two solution. For other closely related results, (see [4–8, 11, 20–22]) and the references cited therein. The study of these equations is challenging and rewarding and is still in its infancy. We believe that the nonlinear rational difference equations are of paramount importance in their own right. Furthermore the results about such equations offer prototypes for the development of the basic theory of the global behavior of nonlinear difference equations.