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A SEMI-ANALYTICAL ANALYSIS OF THE STABILITY OF THE REVERSIBLE SELKOV MODEL

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Abstract. Semi-analytical solutions for the reversible Selkov model are developed and used to investigate its static and dynamic stability. A one-dimensional reaction-diffusion cell is considered with all steps of the Selkov reaction modeled. A coupled set of four partial differential equations is obtained for the precursor, reactant, autocatalyst and final product concentrations. The Galerkin method is applied to approximate the spatial structure of the concentrations to obtain a lower-order, ordinary differential equation model, as an approximation to the governing partial differential equations. The semi-analytical model is analysed to obtain steady-state solutions, bifurcation diagrams and parameter maps in which the different types of bifurcation patterns and Hopf bifurcations occur. The results for this model are compared to the standard (two species) Selkov model. The effect of varying the rate constants, associated with the decay of both the precursor and final product, on the stability of the system is considered in detail. It is shown that increasing these rate constants, and hence the coupling between the reactant/autocatalyst and precursor/final product, stabilizes the system by reducing the area of parameter space in which Hopf bifurcations can occur. The effect of feedback, by varying the various concentrations in the boundary reservoirs, in response to the concentrations in the cell, is also considered. Feedback can be either stabilizing or destabilizing, depending on the sign of the feedback response. The semi-analytical method is shown to generate accurate solutions, by comparison with numerical solutions of the governing partial differential equations.

Keywords. reaction-diffusion equations, reversible Selkov model, singularity theory, Hopf bifurcations, semi-analytical solutions, feedback control.

AMS (MOS) subject classification: 35, 37, 41

1 Introduction

Chemical and biological systems can exhibit oscillatory solutions, multiple steady-state solutions and chaotic behaviours which have been of great interest to both theoreticians and experimentalists for many decades. Some experimental examples of oscillatory behaviour in chemical systems include the Bray-Liebhafsky, Belousov-Zhabotinsky and Briggs-Rausher reactions, for which periodic variations in concentrations can be visualised via changes in colour; see [4] for a review of these reactions and other oscillatory phenomena. Biochemical systems are responsible for many of the oscillations associated with cellular processes such as glycolytic oscillations in yeast, calcium ion