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THE DISCRETE RAMSEY MODEL WITH DECREASING POPULATION GROWTH RATE

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Abstract. This paper extends the Ramsey-Cass-Koopmans growth model of optimal capital accumulation in discrete time by introducing a generic population growth law that satisfies the following properties: population is strictly increasing and bounded, and the population growth rate is decreasing to zero as time tends to infinity. We show that the optimization problem admits a unique solution that can be characterized by the Euler equation. A closed-form solution of the model is presented for the case of a Cobb-Douglas production function and a logarithmic utility function. In contrast to the original model, the solution is not always monotone.

Keywords. Economic growth; Ramsey model; discrete time; decreasing population growth rate; closed-form solution.

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1 Introduction

In the original neoclassical model of economic growth due to Ramsey (1928) and extended by Cass (1965) and Koopmans (1963), it is assumed that the population grows at a constant rate $n > 0$. In discrete time it is natural to define this growth rate as:

$$n = \frac{L_{t+1} - L_t}{L_t}$$

where L_t is the population level at period t, which implies that

$$L_{t+1} = (1 + n) L_t.$$