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GENERIC PERIODICALLY EXPANSIVE VOLUME-PRESERVING DIFFEOMORPHISMS

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Abstract. We show that if C^1-generically, a periodically expansive volume-preserving diffeomorphism is mixing Anosov.

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1 Introduction

In smooth dynamical systems, the notion of expansivity is a main topic of the dynamical systems. It is very close to the stability theory. In fact, Mañé proved that if a diffeomorphism belongs to the C^1-interior of the set of all expansive diffeomorphisms then it is quasi-Anosov. Here, we say that a diffeomorphism f is *quasi-Anosov* if for any $v \in TM(v \neq 0)$ the set $\{\|Df^n(v)\| : n \in \mathbb{Z}\}$ is unbounded. For that, in this paper, we study that a kind of expansivity and hyperbolicity under volume-preserving diffeomorphisms. Let us more precise. Let M be a d-dimensional $(d \geq 2)$ Riemannian closed and connected manifold and let $d(\cdot, \cdot)$ denotes the distance on M inherited by the Riemannian structure. We endow M with a volume-form (cf. [10]) and let μ denote the Lebesgue measure related to it. Let $\mathrm{Diff}_\mu(M)$ denote the set of volume-preserving diffeomorphisms defined on M endowed with the C^1 Whitney topology. The Riemannian inner-product induces a norm $\|\cdot\|$ on the tangent bundle T_xM. We will use the usual uniform norm of a bounded linear map A given by $\|A\| = \sup_{\|v\|=1} \|Av\|$. Let $f \in \mathrm{Diff}_\mu(M)$. We say that f is *expansive* if there is $\alpha > 0$ such that for any pair of distinct points $x, y \in M$, $d(f^n(x), f^n(y)) > \alpha$ for some $n \in \mathbb{Z}$. The number $\alpha > 0$ is called an *expansive constant* for f.

We say that a closed f-invariant set Λ is *hyperbolic* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E^s \oplus E^u$ and there exist constants $C > 0$ and $0 < \lambda < 1$ such that

$$\|D_x f^n|_{E^s_x}\| \leq C\lambda^n \ \text{ and } \ \|D_x f^{-n}|_{E^u_x}\| \leq C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$. If $\Lambda = M$ then f is Anosov.