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ON THE DYNAMICS OF THE NONLINEAR RATIONAL DIFFERENCE EQUATION

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Abstract. In this paper, we discuss some qualitative properties of the positive solutions to the following rational nonlinear difference equation $x_{n+1} = \frac{\alpha x_{n-m}}{\beta+\gamma x_{n-k}x_{n-l}(x_{n-k}+x_{n-l})}$, $n = 0, 1, 2, ...$ where the parameters $\alpha, \beta, \gamma \in (0, \infty)$, while m, k, l are positive integers, such that $m < k < l$. The initial conditions $x_{-m}, ..., x_{-k}, ..., x_{-l}, ..., x_{-1}, ..., x_0$ are arbitrary positive real numbers.

Keywords. Difference equations; Rational difference equations; qualitative properties of solutions of difference equations; Equilibrium points.

AMS (MOS) subject classification: 39A10; 39A11; 39A99;34C99.

1 Introduction

The qualitative study of difference equations is a fertile research area and increasingly attracts many mathematicians. This topic draws its importance from the fact that many real life phenomena are modeled using difference equations. The applications of these difference equations can be found in economy, biology and so on. It is known that nonlinear difference equations are capable of producing a complicated behavior regardless its order. This can be easily seen from the family $x_{n+1} = g_{\mu}(x_n)$, $\mu > 0$, $n \geq 0$. This behavior is ranging according to the value of μ, from the existence of a bounded number of periodic solutions to chaos.

There has been a great interest in studying the global attractivity, the boundedness character and the periodicity nature of nonlinear difference equations. The study of these equations is challenging and rewarding and is still in its infancy. We believe that the nonlinear rational difference equations are of importance in their own right. Furthermore the results about such equations offer prototypes for the development of the basic theory of the global behavior of nonlinear difference equations.
The objective of this article is to investigate some qualitative behavior of the solutions of the nonlinear difference equation.

$$x_{n+1} = \frac{\alpha x_{n-m}}{\beta + \gamma x_{n-k}x_{n-l}\left(x_{n-k} + x_{n-l}\right)}, \quad n = 0, 1, 2, ... \tag{1}$$

where the parameters $\alpha, \beta, \gamma \in (0, \infty)$, while m, k, l are positive integers, such that $m < k < l$. The initial conditions $x_{-m}, ..., x_{-k}, ..., x_{-l}, ..., x_{-1}, ..., x_0$ are arbitrary positive