

## A DIFFERENCE EQUATION WITH ANTI-PERIODIC BOUNDARY CONDITIONS

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**Abstract.** In this paper, we apply an extension of the Leggett-Williams fixed point theorem to the second order difference equation  $\Delta^2 u(k) + f(u(k+1)) = 0$ ,  $k \in \{0, 1, \dots, N\}$ , satisfying the anti-periodic boundary conditions  $u(0) + u(N+2) = 0$ ,  $\Delta u(0) + \Delta u(N+1) = 0$ . Two important results of this paper involve providing the Green's function for  $-\Delta^2 u(k) = 0$  satisfying  $u(0) + u(N+2) = 0$ ,  $\Delta u(0) + \Delta u(N+1) = 0$  and showing this Green's function satisfies a concavity like property. An example is also given.

**Keywords.** Fixed Point Theorem, Difference Equation, Antiperiodic, Antisymmetric, Functional.

**AMS (MOS) subject classification:** 39A10.

### 1. INTRODUCTION

In this paper, we will apply an Avery, Anderson, and Henderson [2] fixed point theorem that is an extension of the Leggett-Williams fixed point theorem [10]. Avery and others have been providing fixed point theorems that relax some of the assumptions of the original Leggett-Williams fixed point theorem; for examples, see [1, 3, 4, 5, 6].

Here, we consider the second order difference equation

$$(1.1) \quad \Delta^2 u(k) + f(u(k+1)) = 0, \quad k \in \{0, 1, \dots, N\},$$

satisfying anti-periodic boundary conditions

$$(1.2) \quad u(0) + u(N+2) = 0, \quad \Delta u(0) + \Delta u(N+1) = 0,$$

where  $f : \mathbb{R} \rightarrow \mathbb{R}$  is continuous and  $\Delta u(k) = u(k+1) - u(k)$  is the forward difference operator. We will show that if  $f$  satisfies certain conditions, then (1.1), (1.2) has an antisymmetric solution in the sense that  $u(N+2-k) = -u(k)$ .

While there has been much work done on ordinary differential equations satisfying antiperiodic boundary values (for some examples, see [7, 8, 9]),