

SEQUENTIAL FRACTIONAL Q -DIFFERENCE EQUATIONS WITH NONLOCAL SUB-STRIP BOUNDARY CONDITIONS

Ravi P. Agarwal^{1,2}, Ahmed Alsaedi², Bashir Ahmad²
and Hana Al-Hutami²

¹Department of Mathematics, Texas A&M University, Kingsville, TX 78363-8202, USA

²Department of Mathematics, Faculty of Science, King Abdulaziz University,
P.O. Box 80203, Jeddah 21589, Saudi Arabia

E-mail: Agarwal@tamuk.edu (RPA), aalsaedi@hotmail.com (AA),
bashirahmad_qau@yahoo.com (BA), hanno.1407@hotmail.com (HA)

Abstract. This paper is devoted to the study of sequential fractional q -difference equations supplemented with nonlocal sub-strip boundary conditions. We obtain some existence results for the given problem by applying standard tools of fixed point theory. The paper concludes with some illustrating examples.

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1 Introduction

In this paper, we consider a boundary value problem of sequential fractional q -difference equations with nonlocal sub-strip boundary conditions given by

$$\begin{cases} {}^c D_q^\alpha ({}^c D_q^\vartheta + \lambda)x(t) = f(t, x(t)), & t \in [0, 1], \quad 0 < q < 1, \\ x(0) = 0, \quad x(\zeta) = a \int_0^\eta x(s) d_q s + b \int_\xi^1 x(s) d_q s, \end{cases} \quad (1)$$

where ${}^c D_q^\alpha$ and ${}^c D_q^\vartheta$ denote the fractional q -derivatives of the Caputo type, $0 < \alpha, \vartheta \leq 1$, f is a given continuous function, $0 < \eta < \zeta < \xi < 1$, $\lambda \neq 0$, $a, b \in \mathbb{R}$. For the significance of nonlocal sub-strip boundary condition, see [1].

Fractional boundary value problems have been investigated by several researchers in recent years. The interest in the subject has been mainly due to the application of fractional calculus in various applied fields. A fractional order differential operator distinguishes itself from a classical differential operator in the sense that it can describe the hereditary properties of various materials and processes. This characteristic of fractional order operators has improved the mathematical modelling of many real world problems of physical and technical sciences [2]-[3]. For some recent work on the topic, we refer the reader to the papers [4]-[8] and the references cited therein.