Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **21 (2014)** 367-388

HYBRID EXTRAGRADIENT METHOD FOR FINDING A COMMON SOLUTION OF THE SPLIT FEASIBILITY AND SYSTEM OF EQUILIBRIUM PROBLEMS

Wiyada Kumam[1], Jitsupa Deepho[2] and Poom Kumam[2,0]

[1]Department of Mathematics and Computer Science,
Faculty of Science and Technology,
Rajamangala University of Technology Thanyaburi (RMUTT)
39, Moo 1, Rangsit-Nakhonnayok Rd., Klong 6, Thanyaburi, Pathumthani 12110,
Thailand

[2]Department of Mathematics, Faculty of Science,
King ongkut's University of Technology Thonburi (KMUTT)
126 Pracha Uthit Rd., Bang Mod, Thrung Khru, Bangkok 10140, Thailand

Abstract. The purpose of this paper is to introduce a new hybrid extragradient iterative algorithm for finding a common element of the set of fixed points of quasi-nonexpansive mappings and satisfying solutions of the split feasibility problem (SFP) and systems of equilibrium problem (SEP) in Hilbert spaces. We prove that the sequence generated by the proposed algorithm converge strongly to a common element of the fixed points set of quasi-nonexpansive mappings, the solution of split feasibility problems and systems of equilibrium problems under mild condition. The results presented in this paper generalized, improve and extend some well-known results in the literature.

Keywords. Extragradient method, Fixed point problems, Hybrid projection method, Split feasibility problems, Systems of equilibrium problems, Qusi-nonexpansive mappings, Lipschitz continuity.

AMS (MOS) subject classification: 47H10, H7H09

[0]Corresponding author: poom.kum@kmutt.ac.th

References

[1] Censor, Y. and Elving, T., 1994, A multiprojection algorithm using Bregman projections in product space, *Numerical Algorithms*, Vol. 8, No. 2-4, pp. 221-239.

[2] Byrne, C., 2002, Iterative oblique projection onto convex subsets and the split feasibility problem, *Inverse Problems*, Vol. 18, No. 2, pp. 441C53.

[3] Xu, H.K., 2010, Iterative methods for the split feasibility problem in infinite-dimensional Hilbert spaces, *Inverse Problems*, Vol. 26, pp. 105018.

[4] Censor, Y., Elving, T., Kopf, N. and Bortfeld, T., 2005, The multiple-set split feasibility problem and its applications for inverse problem, *Inverse Problem*, Vol. 21, No. 6, pp. 2071-2084.

[5] Censor, Y., Motova, A. and Segal, A., 2007, Perturbed projections and subgardient projections for the multiple-set split feasibility problem, *J. Math. Anal. Appl.*, Vol. 327, pp. 1244-1256.

[6] Eicke, B., 1992, Iteration methods for convexly constrained ill-posed problems in Hilbert space, *Numerical Functional Analysis and Optimization*, Vol. 13, No. 5-6, pp. 413-429.

[7] Landweber, L., 1951, An iterative formula for Fredholm integral equations of the first kind, *Amer. J. Math.*, Vol. 73, pp. 615-625.

[8] Mann, W. R., 1953, Mean value methods in iteration, *Proceedings of the American Mathematical Society*, Vol. 4, pp. 506-510.

[9] Dotson Jr, W. G., 1970, On the Mann iterative process, *Transctions of the American Mathematical Society*, Vol. 149, pp. 65-73.

[10] Plubtieng, S., Wangkeeree, R. and Punpaeng, R., 2006, On the convergence of modified Noor iterations with errors for asymptotically nonexpansive mappings, *Journal of Mathematical Analysis and Applicaations*, Vol. 322, No. 2, pp. 1018-1029.

[11] Plubtieng, S. and Wangkeeree, R., 2006, Strong convergence theorems for multi-step Noor iteratins with errors in Banach spaces, *Journal of Mathematical Analysis and Applicaations*, Vol. 321, No. 1, pp. 10-23.

[12] Blum, E. and Oettli, W., 1994, From optimization and variational inequalities to equilibrium problems, *Math. Student*, Vol. 63, pp. 123-145.

[13] Flam, D. S. and Antipin, A. S., 1997, Equilibrium programming using proximal-like algorithms, *Mathematical Programming*, Vol. 78, No. 1, pp. 29-41.

[14] Moudafi, A. and Thera, M., 1999, Proximal and dynamical approches to equilibrium problem, *Lecture note in Economics and Mathematical Systems*, Springer-Verlag, New York, Vol. 477, pp. 187-201.

[15] Takahashi, S. and Takahashi, W., 2007, Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces, *J. Math. Anal. Appl.*, Vol. 331, pp. 506-515.

[16] Takahashi, W. and Zembayashi, K., 2008, Strong convergence theorem by a new hybrid method for equilibrium problems and relatively nonexpansive mappings, *Fixed Point Theory and Applications*, Vol. 2008, Article ID 528476, 11 pages.

[17] Plubtieng, S. and Sriprad, W., 2009, A viscosity approximation method for finding common solutions of variational inclusion, equilibrium problem, and fixed point problem in Hilbert spaces, *Fixed Point Theory and Applications*, Vol. 2009, Article ID 567147, 20 pages.

[18] Plubtieng, S. and Thammathiwat, T., 2010, A viscosity approximation method for equilibrium problems, fixed point problems of nonexpansive mappings and general system of variational inequalities, *Journal of Global Optimization*, Vol. 46, No. 3, pp. 447-464.

[19] Moudafi, A., 2012, From alternating minimization algorithms and systems of variational inequalities to equilibrium problems, *Communications on Applied Nonlinear Analysis*, Vol. 16, No. 3, pp. 31-35.

[20] Yao, Y., Liou, Y.-C. and Yao, J.-C., 20111, New relaxed hybrid extragradient method for fixed point problems, a general system of variational inequality problems and generalized mixed equilibrium problems, *Optimization*, Vol. 60, No. 3, pp. 395-412.

[21] Plubtieng, S. and Sombut, K., 2010, Weak convergence theorems for a system of mixed equilibrium problems and nonspreading mappings in Hilbert spaces, *Journal of Inequalities and Applications*, Vol. 2010, Article ID 246237, 12 pages.

[22] Nakajo, K. and Takahashi, W., 2003, Strong convergence theorems for nonexpansive mappings and nonexpansive semigroup, *J. Math. Anal. Appl.*, Vol. 279, pp. 372-379.

[23] Tada, A. and Takahashi, W., 2007, Weak and strong convergence theorems for a nonexpansive mappings and equilibrium problem, *J. Optim. Theory Appl.*, Vol. 133, pp. 359-370.

[24] Takahashi, W. and Toyoda, M., 2003, Weak convergence theorems for nonexpansive mappings and monotone mappings, *J. Optim. Theory Appl.*, Vol. 133, pp. 417-428.

[25] Phan Tu Vuong, Jean Jacques Strodiot and Van Hien Nguyen, 2012, Extragradient Mrthods and Linesearch Algorithms for Solving Ky Fan Inequalities and Fixed Point Problems," *J. Optim. Theory Appl.*, Vol. 2012, pp. 1-23.

[26] Plubtieng, S. and Sombut, K., 2013, Weak Convergence Theorem for Finding Fixed Point and Solution of Split Feasibility and Systems of Equilibrium Problems, *Abstract and Applied Analysis*, Vol. 2013, Article ID 420409, 8 pages.

[27] Kumam, P., 2009, A new hybrid iterative method for solution of equilibrium problems and fixed point problems for an inverse strongly monotone operator and a nonexpansive mapping, *J. App. Math. Comput.*, Vol. 29, No. 1, pp. 263-280.

[28] Opial, Z., 1967, Weak convergence for successive approximations for nonexpansive mappings, *Bull. Amer. Math. Soc.*, Vol. 73, pp. 591-597.

[29] Geobel, K. and Kirk, W. A., 1990, Topics in Metric Fixed Point Theory, Cambridge Studies in Advanced Mathematics, *Cambridge University Press*, Cambridge, Vol. 28.

[30] Ceng, L.-C. and et al., 2012, An extragradient method for solving split feasibility anf fixed point problems, *Computers and Mathematics with Applications*, Vol. 64, No. 4, pp. 633-642.

[31] Byrne, C., 2004, A unified treatment of some iterative algorithms in signal processing and image reconstruction, *Inverse Problem*, Vol. 20, No. 1, pp. 103-120.

[32] Combettes, P. L., 2004, Solving monotone inclusions via compositions of nonexpansive averaged operator, *Optimization*, Vol. 53, No. 5-6, pp. 475-504.

[33] Marino, G., and Xu, H. K., 2007, Weak and strong convergence theorems for strict pseudo-contractions in Hilbert space, *J. Math. Anal. Appl.*, Vol. 329, pp. 336-346.

[34] Peng, J.-W. and Yao, J.-C., 2008, A new hybrid-extragradient method for generalized mixed equilibrium problems, fixed point problems and variational inequality problems, *Taiwanes Journal of Mathematics*, Vol. 12, No. 6, pp. 1401-1432.

[35] Rockafellar, R. T., 1970, On the maximality of sums of nonlinear monotone operators, *Trans. Amer. Math. Soc.*, Vol. 149, pp. 75-88.

Received May 2014; revised October 2014.

email: journal@monotone.uwaterloo.ca
http://monotone.uwaterloo.ca/∼journal/