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QUENCHING ANALYSIS FOR NONLINEAR VOLTERRA INTEGRAL EQUATIONS

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Abstract. We study nonlinear second-kind Volterra integral equations (VIEs) of Hammerstein type with convolution kernels where the nonlinear kernel function has a singular point. Quenching occurs only when solutions approach the singularity point in finite time, but this is not sufficient for the blow-up of the first derivative at that time. For weakly singular convolution kernels, we give some sharp conditions for the quenching of the solution that are similar to the ones provided in our blow-up analysis for Hammerstein-type VIEs (*J. Integral Equations Appl.* 24 (2012), 487-512). However, for smooth convolution kernels we require new techniques for analyzing the blow-up of the first derivative of the solution. Finally, we apply our quenching conditions to some VIEs that correspond to applications modelled by parabolic differential equations that are driven by moving concentrated nonlinear sources, nonlocal effects or nonlinear boundary conditions.

Keywords. Volterra integral equations; Ordinary differential equations; Parabolic equations; Quenching behaviors; Blow-up.

AMS (MOS) subject classification: 45D05, 45G10.

References

[1] A. Acker and W. Walter, The quenching problems for nonlinear parabolic differential equations, in *Ordinary and partial differential equations* (Dundee, 1976), (W. M. Everitt, B. D. Sleeman, ed.), pp. 1-12, *Lecture Notes in Mathematics* 564, Springer-Verlag, New York, 1976.

[2] A. Acker and B. Kawohl, Remarks on quenching, Nonlinear Anal., 13 (1989), pp. 53-61.

[3] T. K. Boni, On quenching of solutions for some semilinear parabolic equations of second order, Bull. Belg. Math. Soc., 7 (2000), pp. 73-95.

[4] H. Brunner, *Collocation Methods for Volterra Integral and Related Functional Equations*, Cambridge Monographs on Applied and Computational Mathematics, Vol 15, Cambridge, Cambridge University Press, 2004.

[5] H. Brunner and Z.W. Yang, Blow-up behavior of Hammerstein-type Volterra integral equations, J. Integral Equations Appl., 24 (2012), pp. 487-512.

[6] C.Y. Chan and M.K. Kwong, Quenching phenomena for singular nonlinear parabolic equations, Nonlinear Analysis, 12 (1988), pp. 1377-1383.

[7] C.Y. Chan and H.G. Kaper, Quenching for semilinear singular parabolic problems, SIAM J. Math. Anal., 20 (1989), 558-566.

[8] K. Deng and H.A. Levine, On the blowup of u_t at quenching, Proc. Amer. Math. Soc., 106 (1989), 1049-1056.

[9] K. Deng and C.A. Roberts, Quenching for a diffusive equation with a concentrated singularity, Differential Integral Equations, 10 (1997), pp. 369-379.

[10] K. Deng and M.X. Xu, Quenching for a nonlinear diffusion equation with a singular boundary condition, Z. Angew. Math. Phys., 50 (1999), pp. 574-584.

[11] K. Deng and C.L. Zhao, Quenching versus blow-up, J. Partial Differential Equations, 13 (2000), pp. 243-252.

[12] L.C.F. Ferreira and O.S. de Queiroz, A singular parabolic equation with logarithmic nonlinearity and L^p-initial data, J. Differential Equations, 249 (2010), pp. 349-365.

[13] M. Fila and J. Hulshof, A note on the quenching rate, Proc. Amer. Math. Soc., 112 (1991), pp. 473-477.

[14] M. Fila and H.A. Levine, Quenching on the boundary, Nonlinear Anal. TMA, 21 (1993), pp. 795-802.

[15] J.S. Guo, On the quenching behavior of the solution of a semilinear parabolic equation, J. Math. Anal. Appl., 151 (1990), pp. 58-79.

[16] H. Kawarada, On solutions of initial-boundary problem for $u_t = u_{xx} + \frac{1}{1-u}$, Publ. RIMS, Kyoto Univ., 10 (1975), 729-736.

[17] B. Kawohl, Remarks on quenching, Doc. Math., 1 (1996), pp. 199-208.

[18] C.M. Kirk and C.A. Roberts, A quenching problem for the heat equation, J. Integral Equations Appl., 14 (2002), pp. 53-72.

[19] C.M. Kirk and C.A. Roberts, A review of quenching results in the context of nonlinear Volterra integral equations, Dynamics Cont. Discrete Impulsive Systems, Ser. A Math. Anal., 10 (2003) pp. 343-356.

[20] H.A. Levine and J.T. Montgomery, The quenching of solutions of some nonlinear parabolic equations, SIAM J. Math. Anal., 11 (1980), pp. 726-739.

[21] H.A. Levine, The quenching of solutions of linear parabolic and hyperbolic equations with nonlinear boundary conditions, SIAM J. Math. Anal., 14 (1983), pp. 1139-1153.

[22] H.A. Levine, The phenomenon of quenching : A survey, in *Trends in the Theory and Practice of Nonlinear Analysis* (V. Lakshmikantham ed.), pp. 275-268, North Holland, Amsterdam, 1985.

[23] H.A. Levine, Quenching, nonquenching, and beyond quenching for solution of some parabolic equations, Ann. Mat. Pura Appl., 155 (1989), pp. 243-260.

[24] N. Minorsky, *Introduction to Nonlinear Mechanics*, J. W. Edwards, Ann. Arbor, MI, 1947.

[25] C.L. Mu, S.M. Zhou and D.M. Liu, Quenching for a reaction-diffusion system with logarithmic singularity, Nonlinear Anal., 71 (2009), pp. 5599-5605.

[26] W.E. Olmstead and C.A. Roberts, Quenching for the heat equation with a nonlocal nonlinearity, in Nonlinear Problems in Applied Mathematics, SIAM, Philadelphia, PA, 1995, pp. 199-205.

[27] W.E. Olmstead and C.A. Roberts, Critical speed for quenching : Advances in quenching, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal., 8 (2001), pp. 77-88.

[28] C.A. Roberts, Analysis of explosion for nonlinear Volterra equations, J. Comput. Appl. Math., 97 (1998), pp. 153-166.

[29] C.A. Roberts and W.E. Olmstead, Local and non-local boundary quenching, Math. Methods Appl. Sci., 22 (1999), pp. 1465C1484.

[30] C.A. Roberts, Recent results on blow-up and quenching for nonlinear Volterra equations, J. Comput. Appl. Math., 205 (2007), pp. 736-743.

[31] T. Salin, On quenching with logarithmic singularity, Nonlinear Anal., 52 (2003), pp. 261-289.

[32] T. Salin, Quenching-rate estimate for a reaction diffusion equation with weakly singular reaction term, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal., 11 (2004), pp. 469-480.

[33] J.J. Stoker, *Nonlinear Vibrations*, Interscience, New York, 1950.

[34] W. Walter, Parabolic differential equations with a singular nonlinear term, Funkcial. Ekvac., 19 (1976), pp. 271-277.

[35] R.Z. Xu, C.Y. Jin, T. Yu and Y.C. Liu, On quenching for some parabolic problems with combined power-type nonlinearities, Nonlinaer Anal. Real World Appl., 13 (2012), pp. 333-339.

[36] Z.W. Yang and H. Brunner, Blow-up behavior of collocation solutions to Hammerstein-type Volterra integral equations, SIAM J. Numer. Anal., 51 (2013), pp. 2260-2282.

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