

ON THE DYNAMICS OF A SYSTEM OF RATIONAL DIFFERENCE EQUATIONS

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Abstract. The aim of this work is to investigate the global behavior for the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + a}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + b}, \quad n = 0, 1, 2, \dots$$

where the parameters a and b are positive real numbers and the initial conditions x_{-1}, x_0, y_{-1} and y_0 are nonnegative real numbers.

Keywords. Difference Equations, Asymptotic Stability, Periodicity.

AMS (MOS) subject classification: 39A10, 39A11.

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Received February 2014; revised November 2014.

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