

## CLOSED FORM SOLUTIONS OF SOME SYSTEMS OF RATIONAL DIFFERENCE EQUATIONS IN TERMS OF FIBONACCI NUMBERS

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**Abstract.** In this paper we deal with the form of the solutions of the two following systems of rational difference equations

$$x_{n+1} = \frac{y_n(x_{n-2} + y_{n-3})}{y_{n-3} + x_{n-2} - y_n}, y_{n+1} = \frac{x_{n-1}(x_{n-1} + y_{n-2})}{2x_{n-1} + y_{n-2}}$$
$$x_{n+1} = \frac{(y_{n-3} - x_{n-2})y_n}{y_{n-3} - x_{n-2} + y_n}, y_{n+1} = \frac{(y_{n-2} - x_{n-1})x_{n-1}}{y_{n-2}}$$

where  $n \in \mathbb{N}_0$  and the initials conditions are arbitrary nonzero real numbers.

**Keywords.** Periodic solutions; systems of difference equations; Fibonacci numbers.

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