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GENERALIZED CAUCHY PROBLEM INVOLVING A CLASS OF DEGENERATE FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. Our aim is to study a generalized Cauchy problem involving a class of degenerate fractional differential equations in Banach spaces. The existence and stability results are obtained by using the fixed point theory for condensing maps and fractional calculus. An application to fractional partial differential equations is given to illustrate our results.

Keywords. Asymptotic stability; Degenerate fractional differential equation; Impulsive condition; Nonlocal condition; Condensing map; Fixed point; Measure of non-compactness; MNC-estimate.

AMS (MOS) subject classification: 35B35, 35R12, 47H08, 47H10

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