

EXISTENCE OF PERIODIC SOLUTIONS FOR A CLASS OF SECOND-ORDER SUPERQUADRATIC DELAY DIFFERENTIAL EQUATIONS

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Abstract. Using critical point theory (in particular a Linking theorem) we study the existence of periodic solutions for the second-order delay differential equations

$$x''(t) = -f(t, x(t - \tau)),$$

where $f(t, x)$ depends periodically on t and $F(t, x)$ is superquadratic (here $\nabla_x F = f$). In particular we consider the case when f does not satisfy the Ambrosetti-Rabinowitz growth condition.

Keywords. Delay differential equations; Critical point theory; Linking theorem; Superquadratic growth condition.

AMS (MOS) subject classification: 34K13; 34K18; 58E50.

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Received February 2014; revised August 2014.

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