EXISTENCE AND STABILITY OF SYNCHRONOUS AND MIRROR-REFLECTING EQUILIBRIA OF A NEURAL NETWORK

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Abstract. We consider a network of three identical neurons with delayed output. The model for such a network is a system of nonlinear delay differential equations, consisting of multiple discrete delays. Our goal in this article is to establish the existence, and to give a detailed account of the linear stability, of synchronous and mirror-reflecting equilibria.

Keywords. delay differential equations, discrete delay, equilibria, linear stability.

AMS (MOS) subject classification: 34K15, 58F36, 92C20.

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