DYNAMICS OF THE RATIONAL DIFFERENCE EQUATION

\[ x_{n+1} = Ax_n + Bx_{n-k} + Cx_{n-l} + \frac{bx_n x_{n-k} x_{n-l}}{dx_{n-k} - ex_{n-l}} \]

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Abstract. In this article, we study the periodicity, the boundedness and the global stability of the positive solutions of the following nonlinear difference equation

\[ x_{n+1} = Ax_n + Bx_{n-k} + Cx_{n-l} + \frac{bx_n x_{n-k} x_{n-l}}{dx_{n-k} - ex_{n-l}}, \quad n = 0, 1, 2, \ldots \]

where the coefficients \(A, B, C, b, d, e\in(0,\infty)\), while \(k\) and \(l\) are positive integers. The initial conditions \(x_{-1}, \ldots, x_{-k}, \ldots, x_{-1}, x_0\) are arbitrary positive real numbers such that \(k < l\). Some numerical examples will be given to illustrate our results.

Keywords. Difference equations, prime period two solution, boundedness character, locally asymptotically stable, global attractor, global stability.

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