

GLOBAL BEHAVIOR TEST FOR A NONLINEAR DIFFERENCE EQUATION WITH A PERIOD-TWO COEFFICIENT

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Abstract. In this paper, we investigate the boundedness character, the periodic character and the global behavior of positive solutions of the difference equation

$$x_{n+1} = A_n + \frac{x_{n-k}^p}{x_n^p}, \quad n = 0, 1, \dots,$$

where $k \in \mathbb{N}$ and $\{A_n\}$ is a two periodic sequence of nonnegative real numbers and the initial conditions x_{-k}, \dots, x_0 are arbitrary positive real numbers.

Keywords. Boundedness character, dynamics, global stability, non-autonomous, periodicity.

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