

**STRONG CONVERGENCE THEOREMS BASED ON  
A NEW MODIFIED EXTRAGRADIENT METHODS  
FOR FINITE GENERALIZED KY FAN  
INEQUALITIES, VARIATIONAL INEQUALITY  
PROBLEMS AND FIXED POINT PROBLEMS**

Gang Cai

College of Mathematics Science  
Chongqing Normal University, Chongqing 401331, China  
Email: caigang-aaaa@163.com

**Abstract.** In this paper, we introduce a new iterative algorithm for finding a common element of the set of solutions of finite generalized Ky Fan inequalities and the set of solutions of more general variational inequality for finite relaxed cocoercive operators and the set of common fixed points of a nonexpansive semigroup in Hilbert space. The results obtained in this paper improve and extend the results announced by many others.

**Keywords.** Ky Fan inequality; Variational inequality; Fixed point; Nonexpansive semigroup; Hilbert space.

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## References

- [1] E. Blum, W. Oettli, From optimization and variational inequalities to equilibrium problems, *Math. Stud.*, 63, (1994) 123-145.
- [2] F. E. Browder, Convergence of approximants to fixed points of nonexpansive nonlinear mappings in Banach spaces, *Arch. Ration. Mech. Anal.*, 24, (1967) 82-89.
- [3] L. C. Ceng, C. Y. Wang, J. C. Yao, Strong convergence theorems by a relaxed extragradient method for a general system of variational inequalities, *Math. Method Oper. Res.*, 67,(2008) 375-390.
- [4] Y. J. Cho, X. Qin, J. I. Kang, Convergence theorems based on hybrid methods for generalized equilibrium problems and fixed point problems, *Nonlinear Analysis*, 71, (2009) 4203-4214.
- [5] S. S. Chang, H. W. Joseph Lee, C. K. Chan, A new method for solving equilibrium problem fixed point problem and variational inequality problem with application to optimization, *Nonlinear Anal.*, 70, (2009) 3307-3319.
- [6] F. Cianciaruso, G. Marino, L. Muglia, Iterative methods for equilibrium and fixed problems for nonexpansive semigroups in Hilbert spaces, *J. Optim. Theory. Appl.*, 146, (2010) 491-509.
- [7] V. Colao, G. Marino, H. K.Xu, An iterative method for finding common solutions of equilibrium and fixed point problems, *J. Math. Anal. Appl.*, 344, (2008) 340-352.
- [8] K. Fan, A minimax inequality and applications, In: Shisha, O. (ed.) *Inequality III*, pp. 103-113. Academic Press, New York (1972)
- [9] H. Iiduka, W. Takahashi, Strong convergence theorems for nonexpansive mappings and inverse-strongly monotone mappings, *Nonlinear Anal.*, 61, (2005) 341-350.
- [10] C. Jaiboon, P. Kumam, U. W. Humphries, Weak convergence theorem by an extragradient method for variational inequality, equilibrium and fixed point problems, *Bull. Malays. Math. Sci. Soc.*, (2) 32(2), (2009) 173-185.
- [11] C. Jaiboon, P. Kumam, U. W. Humphries, An extragradient method for relaxed cocoercive variational inequality and equilibrium problems, *Anal. Theory Appl.*, 25, (2009), 381-400.
- [12] C. Jaiboon, W. Chantarangsi, P. Kumam, A Convergence theorem based on a hybrid relaxed extragradient method for generalized equilibrium problems and fixed point problems of a finite family of nonexpansive mappings, *Nonlinear Analysis: Hybrid Systems*, 4,(2010) 199-215.
- [13] C. Jaiboon, P. Kumam, U. W. Humphries, T. Ibaraki, Weak and strong convergence theorems by an extragradient method for variational inequality, equilibrium and fixed point problems, *Proceedings of the Asian Conference on Nonlinear Analysis and Optimization (NAO-Asia2008)*. pp. 87-95.
- [14] W. Kumam, P. Kumam, Hybrid iterative scheme by relaxed extragradient method for solutions of equilibrium problems and a general system of variational inequalities with application to optimization, *Nonlinear Analysis: Hybrid Systems*, 3, (2009), 640-656.
- [15] P. Kumam, Strong Convergence Theorems by an Extragradient Method for Solving Variational Inequality and Equilibrium Problems in a Hilbert space, *Turk. J. Math.* 33, (2009) 85-98.
- [16] G. Marino, H. K. Xu, A general iterative method for nonexpansive mappings in Hilbert spaces, *J. Math. Anal. Appl.*, 318, (2006) 43-52.
- [17] M. A. Noor, K. I. Noor, Resolvent methods for solving system of general variational inclusions, *J. Optim. Theory. Appl.*, 148, (2011) 422-430.
- [18] Z. Opial, Weak convergence of the sequence of successive approximation for nonexpansive mapping, *Bull. Amer. Math. Soc.*, 73, (1967) 561-597.

- [19] S. Phiangsungnoen, P. Kumam, A Hybrid Extragradient Method for Solving Ky Fan Inequalities, Variational Inequalities and Fixed Point Problems, Proceedings of the International MultiConference of Engineers and Computer Scientists 2013, Vol II, IMECS 2013, March 13-15, 2013, Hong Kong, pp. 1042-1047.
- [20] S. Plubtieng, T. Thammathiwat, A viscosity approximation method for equilibrium problems, fixed point problems of nonexpansive mappings and a general system of variational inequalities, J. Glob. Optim, 46, (2010) 447-464.
- [21] X. Qin, Y. J. Cho, S. M. Kang, Convergence theorems of common elements for equilibrium problems and fixed point problems in Banach spaces, J. Comput. Appl. Math, 225, (2009) 20-30.
- [22] X. Qin, M. Shang, Y. Su, Strong convergence of a general iterative algorithm for equilibrium problems and variational inequality problems, Math. Comput. Model, 48, (2008) 1033-1046.
- [23] X. Qin, Y. J. Cho, S. M. Kang, Viscosity approximation methods for generalized equilibrium problems and fixed point problems with applications, Nonlinear Anal, 72, (2010) 99-112.
- [24] X. Qin, L-J. Lin, S. M. Kang, On a generalized Ky Fan inequality and asymptotically strict pseudocontractions in the intermediate sense, J. Optim. Theory. Appl, 150, (2011) 553-579.
- [25] R. T. Rockafellar, On the maximality of sums of nonlinear monotone operators, Trans. Amer. Math. Soc, 149, (1970) 75-88.
- [26] T. Suzuki, Strong convergence of Krasnoselskii and Mann's type sequences for one-parameter nonexpansive semigroups without Bochner integrals, J. Math. Anal. Appl, 305, (2005) 227-229.
- [27] S. Takahashi, W. Takahashi, Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces, J. Math. Anal. Appl, 331, (2007) 506-515.
- [28] H. K. Xu, Iterative algorithms for nonlinear operator, J. Lond. Math. Soc, 66, (2002) 240-256.
- [29] Y. Yao, Y. J. Cho, R. Chen, An iterative algorithm for solving fixed point problems, variational inequality problems and mixed equilibrium problems, Nonlinear Anal, 71, (2009) 3363-3373.
- [30] Y. Yao, Y.J.Cho, Y-C. Liou, Algorithms of common solutions for variational inclusions, mixed equilibrium problems and fixed point problems, Euro. J. Oper. Research, 212, (2011) 242-250.

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email: journal@monotone.uwaterloo.ca  
<http://monotone.uwaterloo.ca/~journal/>