Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **21 (2014)** 137-148

NUMERICAL ACCURACY OF TWO BENCHMARK MODELS OF WALKING: THE RIMLESS SPOKED WHEEL AND THE SIMPLEST WALKER

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Abstract. The eigenvalues of the Jacobian of the return map are used to quantify the stability of discrete dynamical systems, such as, the rimless wheel and the simplest walker. The accuracy this Jacobian, usually obtained by finite differencing, depends on the step size. Even with the most optimal step size, only moderate accuracy is obtained. Here, we obtain the Jacobian by numerically integrating the gradient of the equations of motion. For the rimless wheel, our eigenvalue estimate is accurate to 12 significant digits and is better than 9 significant digits obtained by finite differencing with optimal step size obtained by Coleman [Dynamics of Continuous, Discrete and Impulsive Systems Series B, 16, 2009]. We first show that our method is able to produce the eigenvalues accurate to 12 significant digits obtained by known analytical solution for the rimless wheel. This benchmark calculation then permits us to make the claim that the eigenvalues of the simplest walker, for which the analytical solution is unknown, obtained using our method are accurate to 12 significant digits.
Keywords. Walking, Simplest walker, Rimless wheel, fixed point, Poincaré return map, stride function, periodic motion, discrete dynamics, intermittent contact, eigen-values.

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Received September 2013; revised April 2014.

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