

**CONVERGENCE OF ITERATES OF UNIFORMLY
 L -LIPSCHITZIAN AND GENERALIZED
ASYMPTOTICALLY NONEXPANSIVE MAPPINGS
IN CAT(0) SPACES**

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Abstract. We study strong and Δ -convergence of a modified S-type iteration process inspired by Agarwal, O'Regal and Sahu (2007) for uniformly L -Lipschitzian and generalized asymptotically nonexpansive mappings in CAT(0) spaces.

Keywords. Strong convergence; Δ -convergence; CAT(0) space; S-type iteration process; Generalized asymptotically nonexpansive mapping.

AMS subject classification: 47H10, 54H25.

References

- [1] R.P. Agarwal, D. O'Regan, D.R. Sahu, Iterative construction of fixed points of nearly asymptotically nonexpansive mappings, *J. Nonlinear Convex Anal.*, **8**, (2007), 61-79.
- [2] M.R. Bridson, A. Haefliger, Metric Spaces of Non-positive Curvature, Grundlehren der Mathematischen Wissenschaften 319, Springer-Verlag, Berlin, Germany, (1999).
- [3] S.S. Chang, L. Wang, H.W. Joseph Lee, C.K. Chan, L. Yang, Demiclosed principle and Δ -convergence theorems for total asymptotically nonexpansive mappings in CAT(0) spaces, *Appl. Math. Comput.*, **219**, (2012), 2611-2617.
- [4] Y.J. Cho, L. Ćirić, S. Wang, Convergence theorems for nonexpansive semigroups in CAT(0) spaces, *Nonlinear Anal.*, **74**, (2011), 6050-6059.
- [5] A. Cuntavepanit, B. Panyanak, Strong convergence of modified Halpern iterations in CAT(0) spaces, *Fixed Point Theory Appl.*, **2011**, (2011), Art. ID 869458, 11 pages, doi:10.1155/2011/869458.
- [6] S. Dhompongsa, W.A. Kirk, B. Sims, Fixed points of uniformly lipschitzian mappings, *Nonlinear Anal.*, **65**, (2006), 762-772.
- [7] S. Dhompongsa, B. Panyanak, On Δ -convergence theorems in CAT(0) spaces, *Comput. Math. Appl.*, **56**, (2008), 2572-2579.
- [8] H. Fukhar-ud-din, Strong convergence of an Ishikawa-type algorithm in CAT(0) spaces, *Fixed Point Theory Appl.*, **2013**, 2013:207.
- [9] K. Goebel, W.A. Kirk, A fixed point theorem for asymptotically nonexpansive mappings, *Proc. Amer. Math. Soc.*, **35**, (1972), 171-174.
- [10] S.H. Khan, M. Abbas, Strong and Δ -convergence of some iterative schemes in CAT(0) spaces, *Comput. Math. Appl.*, **61**, (2011), 109-116.
- [11] A.R. Khan, M.A. Khamsi, H. Fukhar-ud-din, Strong convergence of a general iteration scheme in CAT(0) spaces, *Nonlinear Anal.*, **74**, (2011), 783-791.
- [12] W.A. Kirk, Geodesic geometry and fixed point theory II, in: Proceeding of the International Conference in Fixed Point Theory and Applications, Valencia, Spain, (2003), 113-142.
- [13] W.A. Kirk, B. Panyanak, A concept of convergence in geodesic spaces, *Nonlinear Anal.*, **68**, (2008), 3689-3696.
- [14] T. Laokul, B. Panyanak, Approximating fixed points of nonexpansive mappings in CAT(0) spaces, *Int. J. Math. Anal.*, **3**, (2009), 1305-1315.
- [15] W. Laowang, B. Panyanak, Strong and Δ -convergence theorems for multivalued mappings in CAT(0) spaces, *J. Ineq. Appl.*, **2009**, (2009), Art. Id 730132, 16 pages.
- [16] B. Nanjaras, B. Panyanak, Demiclosed principle for asymptotically nonexpansive mappings in CAT(0) spaces, *Fixed Point Theory Appl.*, **2010**, (2010), Art. ID 268780, 14 pages.
- [17] W. Phuengrattana, S. Suantai, Fixed point theorems for a semigroup of generalized asymptotically nonexpansive mappings in CAT(0) spaces, *Fixed Point Theory Appl.*, **2012**, 2012:230.
- [18] W. Phuengrattana, S. Suantai, Existence theorems for generalized asymptotically nonexpansive mappings in uniformly convex metric spaces, *J. Convex Anal.* (in press).

- [19] S. Saejung, Halpern's iteration in CAT(0) spaces, *Fixed Point Theory Appl.*, **2010**, (2010), Art. Id 471781, 13 pages, doi:10.1155/2010/471781.
- [20] S. Saejung, S. Suantai, P. Yotkaew, A note on "Common fixed point of multistep Noor iteration with errors for a finite family of generalized asymptotically quasi-nonexpansive mappings", *Abstr. Appl. Anal.*, **2009**, (2009), Art. Id 283461, 9 pages, doi:10.1155/2009/283461.
- [21] A. Şahin, M. Başarır, On the strong convergence of a modified S-iteration process for asymptotically quasi-nonexpansive mappings in a CAT(0) space, *Fixed Point Theory Appl.*, **2013**, 2013:12.
- [22] N. Shahzad, H. Zegeye, Strong convergence of an implicit iteration process for a finite family of generalized asymptotically quasi-nonexpansive maps, *Appl. Math. Comput.*, **189**, (2007), 1058-1065.
- [23] L.Y. Shi, R.D. Chen, Y.J. Wu, Δ -convergence problems for asymptotically nonexpansive mappings in CAT(0) spaces, *Abstr. Appl. Anal.*, **2013**, (2013), Art. Id 251705, 6 pages, doi:10.1155/2013/251705.
- [24] J.F. Tang, S.S. Chang, H.W. Joseph Lee, C.K. Chan, Iterative algorithm and Δ -convergence theorems for total asymptotically nonexpansive mappings in CAT(0) spaces, *Abstr. Appl. Anal.*, **2012**, (2012), Art. Id 965751, 11 pages, doi:10.1155/2012/965751.
- [25] K.K. Tan, H.K. Xu, The nonlinear ergodic theorem for asymptotically nonexpansive mappings in Banach spaces, *Proc. Amer. Math. Soc.*, **114**, (1992), 399-404.

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