

ON $(K-NUC)$ -PROPERTY IN MUSIELAK-ORLICZ SPACES DEFINED BY DE LA VALLÉE-POUSSIN MEANS AND SOME COUNTABLY MODULATED SPACES

Atanu Manna¹ and P. D. Srivastava²

¹ ²Department of Mathematics
Indian Institute of Technology Kharagpur, Kharagpur-721302, India

Abstract. A sufficient condition for Musielak-Orlicz sequence space $V_{\Phi}(\lambda)$ defined by de la Vallée-Poussin means to be $(k-NUC)$, $k \geq 2$ is obtained. Under certain assumptions on φ -functions, it is shown that the modular space X_{σ_s} is a Fréchet space and the modular spaces $X_{\hat{\sigma}}$ for $\hat{\sigma} = \sigma_w^{\mu}, \sigma_q^Q, \sigma_0$ are identical as sets.

Keywords. Musielak-Orlicz function, de la Vallée-Poussin means, Luxemburg norm, $(k-NUC)$ -property, Countably modulated spaces.

AMS (MOS) subject classification: 46B20, 46A45, 46A80.

References

- [1] J. Albrycht and J. Musielak, Countably modulated spaces, *Studia Math.* **31**, (1968) 331 – 337.
- [2] A. Baernstein, On reflexivity and summability, *Studia Math.* **42**, (1972) 91 – 94.
- [3] Y. Cui and H. Hudzik, Some geometric properties related to fixed point theory in Cesàro space, *Collect. Math* **50**, (1999) 277 – 288.
- [4] Y. Cui, H. Hudzik, W. Ping, , On k -nearly uniform convexity in Orlicz spaces, *Rev. R. Acad. Cienc. Exact. Fis. Nat (Esp)* **94**, (4) (2000) 461 – 466.
- [5] Y. Cui, H. Hudzik, N. Petrot, S. Suantai, A. Szymaszkiewicz, Basic topological and geometric properties of Cesàro-Orlicz spaces, *Proc. Indian Acad. Sci. (Math. Sci.)*, **115**, 4(2005), 461-476.
- [6] M. Denker and H. Hudzik, Uniformly non- $l_n^{(1)}$ Musielak-Orlicz sequence spaces, *Proc. Indian Acad. Sci.* **101**, (2)(1991), 71 – 86.
- [7] R. Huff, Banach Spaces which are nearly uniformly convex, *Rockey Mountain J. Math.* **10**, (1980) 743 – 749.
- [8] A. Kaminska, Uniform rotundity of Musielak-Orlicz sequence spaces, *Journal of Approximation Theory* **47**, (4) (1986) 302 – 322.
- [9] D. Kutzarova, k - β and k -nearly uniformly convex Banach spaces, *J. Math. Anal. Appl.* **162**, (2)(1991) 322 – 338.
- [10] L. Leindler, Über die verallgemeinerte de la Vallée-Poussinsche Summierbarkeit allgemeiner Orthogonalreihen, *Acta Math. Acad. Sci. Hungar.* **16**, (1965) 375 – 387.
- [11] J. Musielak and A. Waszak, On some countably modulated spaces, *Studia Math.* **38**, (1970) 51 – 57.
- [12] J. Musielak and A. Waszak, Some new countably modulated mpaces, *Comment. Math.* **15**, (1971) 209 – 215.
- [13] J. Musielak, *Orlicz spaces and Modular spaces*, Springer Lecture notes in Math. , Vol. **1034** (Springer, 1983).
- [14] J. Musielak and W. Orlicz, On modular spaces, *Studia Math.* **18**, (1959) 49 – 65.
- [15] J. Musielak and A. Waszak, Sequence spaces generated by moduli of smoothness, *Revista Mathematica de la Universidad Complutense de Madrid* **8**, (1) (1995) 91 – 105.
- [16] W. Orlicz, A note on modular spaces I, *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys*, **9**,(1961), 157-162.
- [17] S. Prus (Eds. W. A. Kirk and B. Sims), *Geometrical background of metric fixed point theory*, Handbook of Metric Fixed Point Theory, Kluwer Academic Publishers, Dordrecht, 2001, 93-132.
- [18] M. M. Rao and Z.D. Ren, *Theory of Orlicz spaces*, Monographs and Textbooks in Pure and Applied Mathematics, Vol. **146** (Marcel Dekker Inc., 1991).
- [19] W. Sanhan, S. Suantai, On k -nearly uniform convex property in generalized Cesàro sequence spaces, *Int. J. Math. Math. Sci.* (57), 2003 3599 – 3607.
- [20] J. S. Shiue, Cesàro sequence spaces, *Tamkang Journal of Mathematics*, **1**, (1970) 19-25.
- [21] N. Şimşek, E. Savaş, V. Karakaya, Some geometric and topological properties of a new sequence spaces defined by De La Vallée-Poussin Mean, *Journal of Computational Analysis and Applications*, **12**, 4(2010) 768-779.
- [22] N. Şimşek, On some geometric of sequence space defined by De La Vallée-Poussin Mean, *Journal of Computational Analysis and Applications*, **13**, 3(2011) 565-573.

- [23] P. D. Srivastava, A. Manna, Some difference sequence spaces generated by de la Vallée-Poussin mean, *Asian-European J. Math.*, **6**, 2(2013) 18 pages.
- [24] J. M. Wang, X. B. Liu and Y. Cui, Local uniform rotundity in Musielak-Orlicz sequence space equipped with Luxemburg norm, *Comment. Math. Prace. Mat* **46**, 1(2006) 131 – 139.
- [25] R. Wangkeeree, On property $(k-NUC)$ in Cesàro-Musielak-Orlicz sequence spaces, *Thai Journal of Mathematics* **1**, (2003) 119 – 130.

Received January 2014; revised April 2014.

email: journal@monotone.uwaterloo.ca
<http://monotone.uwaterloo.ca/~journal/>