

**ANALYSIS OF PIECEWISE-CONTINUOUS
EXTENSIONS OF PERIODIC LINEAR IMPULSIVE
DIFFERENTIAL EQUATIONS WITH FIXED,
STRICTLY INHOMOGENEOUS IMPULSES**

Kevin E.M. Church and Robert J. Smith?

Department of Mathematics and Statistics
University of Ottawa, Ottawa, Canada

Abstract. We introduce the notion of impulse extension equations for linear fixed-time impulsive differential equations (IDEs) with strictly inhomogeneous impulses. These differential equations can be thought of as representing the underlying processes for which such linear fixed-time IDEs are a limiting case. We will establish basic existence and uniqueness results, and then develop sufficient conditions for existence and uniqueness of periodic solutions, and study an example of an IDE whose every solution is periodic, even though almost all impulse extension equations which resembles it admit no periodic or bounded solutions.

Keywords. Impulsive differential equation, impulse extension, mathematical model, periodic solution.

AMS (MOS) subject classification: 34A36, 34A37.

References

- [1] M. Akhmet, *Principles of Discontinuous Dynamical Systems*, Springer [2010].
- [2] D.D. Bainov and P.S. Simeonov, *Impulsive Differential Equations: Periodic Solutions and Applications*, Longman Scientific & Technical, Burnt Mill [1993].
- [3] E.M. Bonotto, and M. Federson, *Limit sets and the Poincaré-Bendixson Theorem in impulsive semidynamical systems*, Journal of Differential Equations, Vol. 244, 2334–2349 [2008].
- [4] A. d’Onofrio, *On pulse vaccination strategy in the SIR epidemic model with vertical transmission*, Applied Mathematics Letters Vol. 18, 729-732 [2005].
- [5] J. Hale, *Ordinary Differential Equations*. Dover [2009].
- [6] R. D. Holt, *Theoretical perspectives on resource pulses*, Ecology, Vol. 89, 671–681 [2008].
- [7] O. Krakovska and L.M. Wahl, *Optimal drug treatment regimens for HIV depend on adherence*, Journal of Theoretical Biology Vol. 246, 499-509 [2007].
- [8] V. Lashmikantham, D.D. Bainov and P.S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific Publishing, Singapore [1989].
- [9] Y. Li, J. Cui, *The effect of constant and pulse vaccination on SIS epidemic models incorporating media coverage*, Communications in Nonlinear Sciences and Numerical Simulations, Vol. 14, 2353–2365 [2009].
- [10] B. Liu, Z. Teng and L. Chen, *The effect of impulsive spraying pesticide on stage-structured population models with birth pulse*, Journal of Biological Systems, Vol. 13, 31–44 [2005].
- [11] Nie, L, et. al. *Qualitative analysis of a modified Leslie-Gower and Holling-type II predator-prey model with state dependent impulsive effect*, Nonlinear Analysis: Real World Applications, Vol. 11, 1364–1373 [2010].
- [12] A.M. Samoilenko and N.A. Perestyuk, *Impulsive Differential Equations*, World Scientific Publishing, Singapore, [1995].
- [13] R.J. Smith? and E.J. Schwartz, *Predicting the potential impact of a cytotoxic T-lymphocyte HIV vaccine: How often should you vaccinate and how strong should the vaccine be?* Mathematical Biosciences, Vol. 212, 180–187 [2008].
- [14] R.J. Smith?, *Adherence to antiretroviral HIV drugs: how many doses can you miss before resistance emerges?* Proceedings of the Royal Society B Vol. 273, 617–624 [2006].
- [15] G. Stamov, *Almost Periodic Solutions of Impulsive Differential Equations*, Springer [2012].
- [16] J.M. Tchuente, N. Dube, C.P. Bhunu, R.J. Smith? and C.T. Bauch, *The impact of media coverage on the transmission dynamics of human influenza*, BMC Public Health 2011, Vol. 11(Suppl 1), S5 [2011].
- [17] D. Wexler, *Solutions périodiques et presque périodiques des systèmes d’équations différentielles aux impulsions*, Revue Roumaine Math. Pures et Appl., Vol. 10, 1163–1199 [1965].
- [18] D. Wexler, *Solutions presque des systèmes linéaires à perturbations distribution*, Revue Roumaine Math. Pures et Appl., Vol. 13, 111—129 [1968].
- [19] Yang et. al, *What can we learn from resource pulses?* Ecology, Vol. 89, 621–634 [2008].

Received February 2013; revised February 2014.

email: journal@monotone.uwaterloo.ca

http://monotone.uwaterloo.ca/~journal/