

STABILITY IN A DISTRIBUTED DELAY DIFFERENTIAL EQUATION

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Abstract. The subject of the paper is the asymptotic stability analysis of the trivial equilibrium of a scalar and linear delay differential equation with two (infinite) distributed delays characterised by general delay kernels. The goal of the article is to determine conditions on the parameters of the equation guaranteeing stability. The results are based on the exact analysis of the characteristic equation of the delay differential equation.

Keywords. delay differential equation; distributed delay; characteristic equation; Lyapunov-Krasovskii functionals; stability; D-subdivision.

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