

NONEXISTENCE OF SOLUTIONS FOR A CLASS OF DEGENERATE OR SINGULAR EQUATIONS

Jinjun Dai¹ and Chunhua Wang²

¹School of Mathematics and Statistics
Central China Normal University, Wuhan, 430079, P. R. China
e-mail: jjdai@mail.ccnu.edu.cn

²School of Mathematics and Statistics
Central China Normal University, Wuhan, 430079, P. R. China
e-mail: chunhuawang@mail.ccnu.edu.cn (Corresponding author)

Abstract. This paper investigates the following degenerate or singular equations

$$-div(|x|^{-2\alpha}\nabla u) = \frac{A}{|x|^{2(\alpha+a)}}u + \frac{u^\theta}{|x|^{b\theta}}, \quad u(x) > 0, \quad x \in \mathbb{R}^n \setminus \{0\},$$

where $a, b \in \mathbb{R}$, $n > 2$, $A \in \mathbb{R}$, $\alpha > 0$ and $\theta > 0$, $p = \frac{2n}{n-2(1+a-b)}$. In various range of the parameters involved, we mainly obtain nonexistence of positive solutions by combining the Kelvin transform, methods of moving planes and moving spheres and classical arguments on ODE's.

Keywords. Degenerate or singular equations; Radial positive solutions; Kelvin transform; Moving plane methods; Moving sphere methods.

AMS (MOS) subject classification: 35A01, 35B45, 35J15.

References

- [1] T. Bartsch, S. Peng and Z. Zhang, Existence and non-existence of solutions to elliptic equations related to the Caffarelli-Kohn-Nirenberg inequalities, *Calc. Var. PDE.*, **30**, (2007)113-136.
- [2] K. Chou and C. Chu, On the best constant for a weighted Sobolev-Hardy inequality, *J. London Math. Soc(2)*, **48**, (1993)137-151.
- [3] M. Conti, S. Crotti and D. Pardo, On the existence of positive solutions for a class of singular elliptic equations, *Adv. Differential Equations*, **3**, (1998)111-132.
- [4] D. Cao and P. Han, Solutions for semilinear elliptic equations with critical exponents and Hardy potential, *J. Diff. Equats.*, **205**, (2004)521-537.
- [5] L. Caffarelli, R. Kohn and L. Nirenberg, First order interpolation inequalities with weighes, *Compositio Mathematica*, **53**, (1984)259-275.
- [6] W. Chen and C. Li, Classification of solutions of nonlinear elliptic equations, *Duke Math. J.*, **63**, (1991)615-623.
- [7] D. Cao and S. Peng, A global compactness result for singular elliptic problems involving critical Sobolev exponent, *Proc. Amer. Math. Soc.* **131**, (2003)1857-1866.
- [8] D. Cao and S. Peng, A note on the sign-changing solutions to elliptic problems wiyh critical Sobolev exponent and Hardy terms, *J. Diff. Equats.*, **193**, (2003)424-434.
- [9] F. Catrina and Z. Q. Wang, On the Caffarelli-Kohn-Nirenberg inequalities: sharp constants, existence(and nonexistence), and symmetry of external functions, *Comm. Pure Appl. Math.*, **54**, (2001)229-258.
- [10] R. Dautray and J. L. Lions, Mathematical analysis and numerical methods for science and technology, Physical origins and classical methods. Springer-Verlag, Berlin, 1990.
- [11] H. Egnell, Elliptic boundary value problems with singular coefficients and critical nonlinearities, *Indiana Univ. Math. J.*, **38**, (1989)235-251.
- [12] Y. Ehibara, Y. Furusho and T. Senba, Dirichlet problem for a semilinear elliptic equation with singular coefficient,*Nonlinear Anal. TMA.* **15**, (1990)299-306.
- [13] A. M. Fink, J. A. Gatica, G. E. Hernandez and P. Waltman, Approximation of solutions of singular second-order boundary value problems, *SIAM J. Math. Anal.* **22**, (1991)440-462.
- [14] C. Li, Monotonicity and symmetry of solutions of fully nonlinear elliptic equations on unbounded domains, *Comm. Part. Eq.* **16**, (1991) 585-615.
- [15] G. Li and S. Peng, Remarks on elliptic problems involving the Caffarelli-Kohn-Nirenberg inequalities, *Proc. Amer. Math. Soc.*, **136**, (2008)1221-1228.
- [16] M. Struwe, Variational methods and applications to nonlinear partial differential equation and Hamiltonian systems, Springer-Verlag, Berlin, 1990.
- [7] S. Terracini, On positive solutions to a class equations with a singular coefficient and critical exponent, *Adv. Diff. Equats.*, **2**, (1996)241-264.
- [18] S. Tai and C. Wang, Existence of positive solutions for a class of degenerate or singular equations, *Nonlinear Anal. TMA.*, **71**, (2009)1691-1698.
- [19] Z. Q. Wang and M. Willem, Singular minimization problems,*J. Diff. Equats.*, **161**, (2000)307-320.
- [20] X. Zhu and J. Yang, Regularity for quasilinear elliptic equations involving critical Sobolev exponent, *J. Systems Sci. Math. Sci.*, **9**, (1988)47-52.

Received November 2012; revised February 2014.

email: journal@monotone.uwaterloo.ca

<http://monotone.uwaterloo.ca/~journal/>